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To: EDGES Group  
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 Subject: Effects of digitizer non linearity

In memo #36 it was shown that digitizer non-linearity produces errors which are centered on the sources of RFI. For example, a spectrometer using 3-position switching of a 5 MHz bandpass stepped from 100 to 200 MHz will have systematics introduced whenever there is RFI within a 5MHz section. This means that even at the quietest sites a 5 MHz band centered on the 137-138 MHz Orbcomm band will be corrupted. In a wideband sampled system the non linearity corrupts the entire spectrum but it does so in a manner that may be acceptable. First the non-linearity produces harmonics and aliased harmonics of the RFI as well as intermodulation products between different sources of narrowband RFI. Second the non-linearity corrupts the calibrated spectrum in a manner that varies with the smoothness of the bandpass. From a simplified analysis:

$$T_{cal} \left[ \frac{p_0 - p_1}{p_2 - p_1} \right] + T_L = T_A - \alpha T_A + \beta T_A / g$$

where  $p_0$ ,  $p_1$  and  $p_2$  are the power spectra on antenna, load and load plus calibrator respectively.  $\alpha$  and  $\beta$  are small positive constants which account for the non-linearity.

For a 10% non-linearity at RFI power 26 dB over the noise total power

$$\alpha \sim 0.2$$

$$\beta \sim 0.01$$

$\alpha$  scales with the non-linearity and  $\beta$  scales as the non-linearity squared.

The linearity is defined as peak deviation from linear

$$\text{e.g. a sine function } \sim x - x * x * x / 6 = x(1 - x * x / 6)$$

when  $x = 0.24x * x / 6 = 0.01$  1% non-linear

$\alpha$  distorts the spectrum – but only by rescaling it

$\beta = 1e-4$  for 1% non-linearity from the RFI and scales with the non-linearity squared.

The term in  $\beta$  is a more serious problem as it depends inversely on the bandpass  $g$  (with  $g=1$  for a constant bandpass). If the bandpass is smooth and is well fit by the polynomial used to fit to  $T_a$  then the effect is no different than  $\alpha$ .

If a ADC has THD – total harmonic distortion below -50 dB the non-linearity should be better than 0.5% and the systematics should be very low. For example if the bandpass has 1 dB ripple not fit by the polynomial and the non-linearity is 0.5 % then the  $\beta$  term gives

$$\beta * (1 / g) * T_a = 2.5e - 5 * 0.25 * 1e3 \sim 6mK$$

For a bandpass ripple of 2 dB peak to peak and a Ta of  $10^3$ K I obtained the following results from simulations using the gsl-rng random number generator to generate Gaussian signals.

Non-linearity	Rms deviation
14%	10K
8%	2K

Figure 1 shows the spectrum of Ta for 8% non-linearity on a log scale and figure 2 shows the spectrum after removing the simulated RFI and fitting a 7 term polynomial to a frequencies from 60 to 180 MHz. Figure 3 shows the spectrum for a 1% non-linearity after fitting a polynomial.

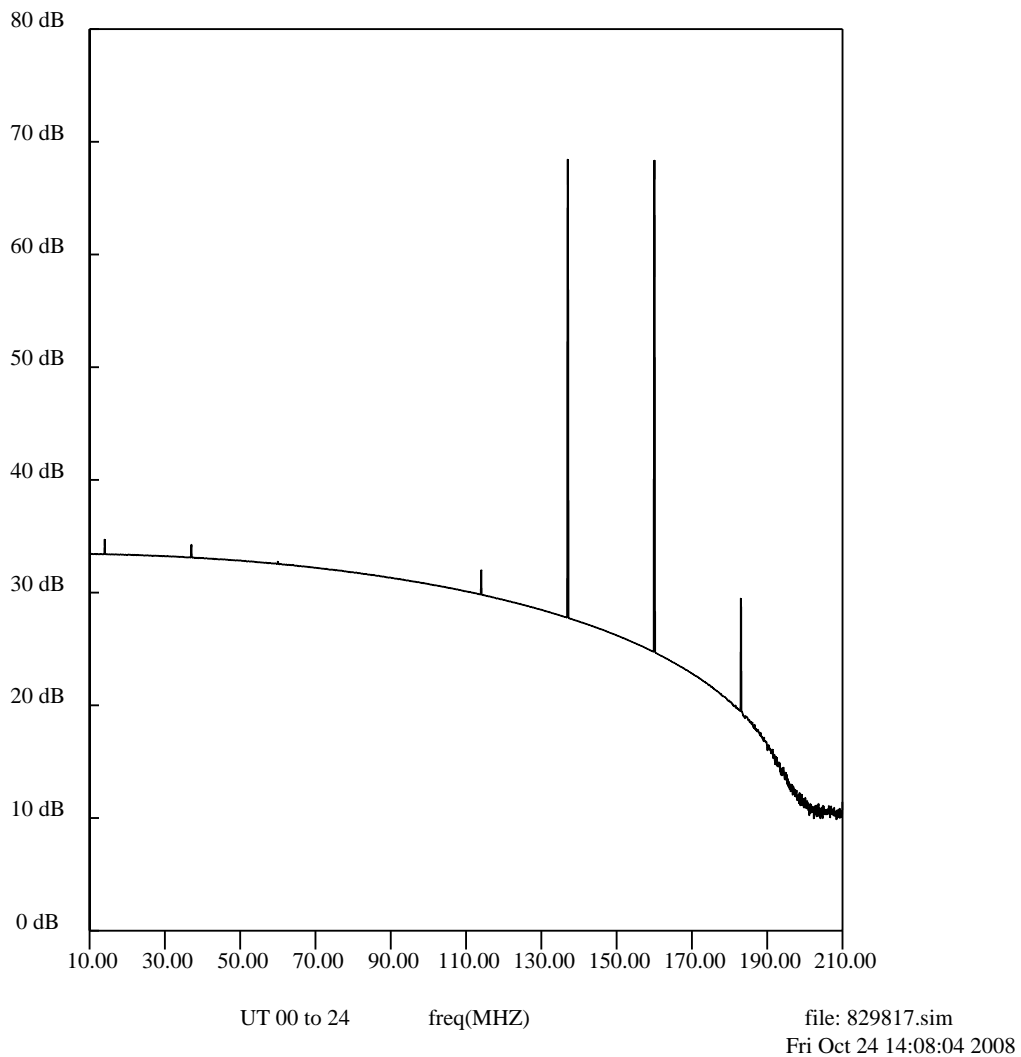


Figure 1.

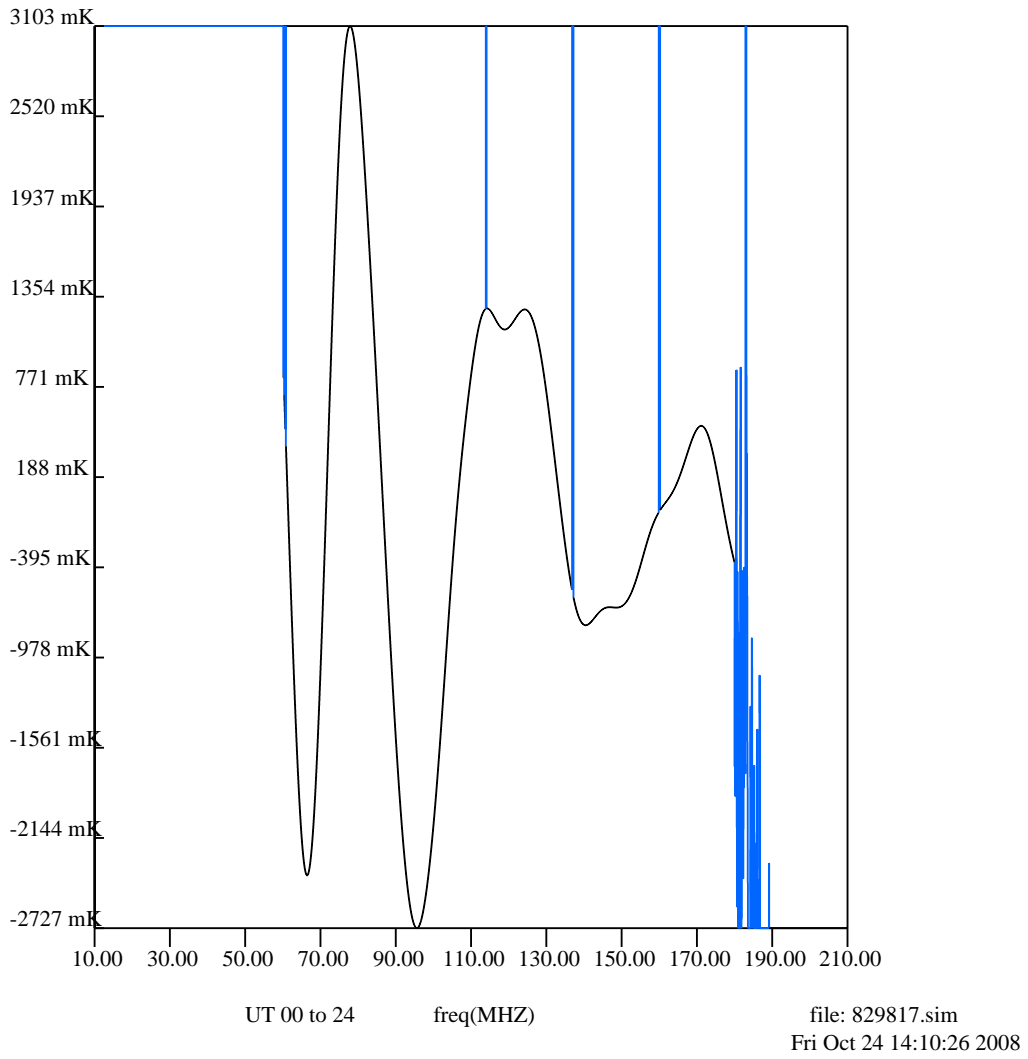


Figure 2.

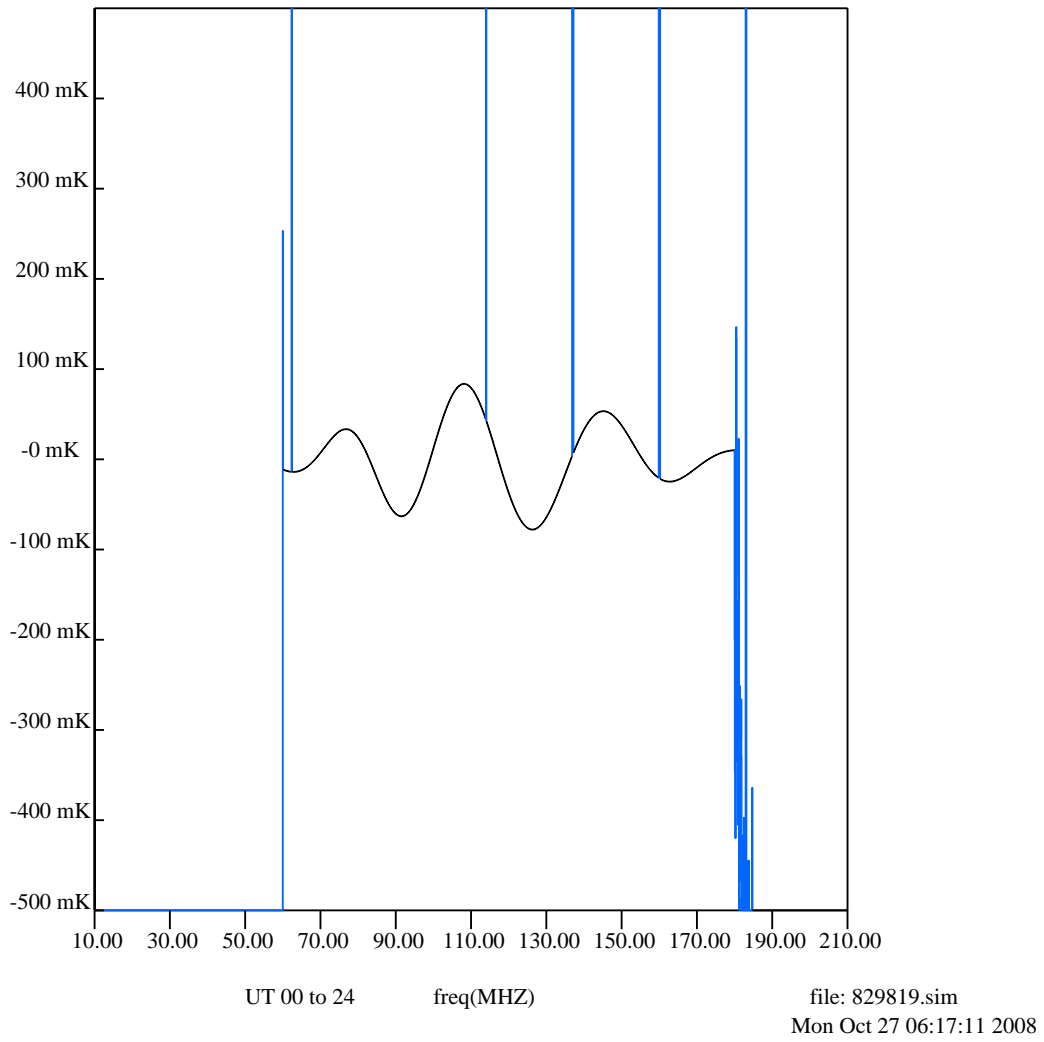


Figure 3.