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To: EDGES Group
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Subject: Simulation of performance with LNA calibration

Simulations have been made with the following assumptions:

1. The antenna reflection coefficient (or equivalently impedance) has been measured
2. The 3-position switching can be modeled using the antenna reflection coefficient data and the LNA noise model. [The modeling can be tested and refined by connecting an open ended low lows cable of known loss and electrical length.]
3. Separate observations are made of the sky with the majority of the Galactic plane below and above the horizon.
4. The spectral index of the sky is constant

Under the assumptions above

$$T_u(\nu) = \alpha T_{sky}(\nu) f(\nu) + T_{LNA}(\nu) \quad (1)$$

$$T_D(\nu) = T_{sky}(\nu) f(\nu) + T_{LNA}(\nu) \quad (2)$$

Where T_u and T_D are the 3-position switch calibrated spectra from Galaxy “up” and “down” respectively. $f(\nu)$ is the corruption of the spectrum due to the effect of antenna mismatch on the sky noise. $T_{LNA}(\nu)$ is the contribution due to the antenna mismatch to the LNA noise. α is a constant.

From (1) and (2) above

$$T_{LNA}(\nu) = \left(\alpha T_D(\nu) - T_u(\nu)\right) / (\alpha - 1) \quad (3)$$

The table below shows in the rms of the residuals after the best fit 5 term polynomial is subtracted. T_{sky} is assumed to be $150(\alpha/150)^{-2.5}$ and $\alpha = 2$. Antenna data for the frequency range 80-160 MHz was taken from EZNEC.

Rms residual mK	Conditions
0	Zero noise LNA or perfect LNA model
76	LNA cooled to 77 K no LNA model
45	Room temperature LNA of current design
16	Improved LNA

374	Without LNA noise model
5054	Without "Galactic calibration"

Table 1. Simulations over 2:1 frequency range.

A complete model of the delectability of the EoR signature is made as follows:

1. Antenna impedance data calculated using NEC to simulate actual measurement using a portable network analyzer.
2. Fit a polynomial to the real and imaginary components of the antenna impedance.
3. Assume a sky temperature input plus EoR signature and noise

$$T_{sky} = 300\left(\nu/150\right)^{-2.5} + EoR\left(\nu\right) + noise\left(\nu\right)$$

4. Use the LNA model to calculate the LNA output looking at the antenna, a 50ohm load and a 50ohm load plus calibrated noise.
5. Derive the 3-position switched calibrated output for observations with Galaxy up and Galaxy down. For "Galaxy up" assume 600 K at 150 MHz.
6. Derive the "EoR model" by subtracting a model based on an input without EoR signature or noise and circuit model parameters perturbed by a fixed percentage. Also derive a second "EoR model" using the "Galactic Calibration" scheme by taking twice the "Galaxy down" spectrum minus the "Galaxy up" spectrum.
7. Fit the EoR models with a polynomial plus the expected EoR signature:

$$EoR\left(\nu\right) = \left(\frac{27e-3}{2}\right)\left(\frac{1+z_0}{10}\right)^{1/2} tanh\left(z - z_0/dz\right)$$

The probability of detection is determined by scanning the EoR frequency added to the input and counting the fraction of the times the correct EoR frequency is correctly detected in a search for the EoR frequency in the fit.

Delta_z	Cal method	antenna rfl.(dB)	swid(MHz)	Cal(%)	noise(mK)	prob(%)	poly
0.1	G	-8	20	5	2	60	2
0.1	M	-8	20	5	2	70	5
0.1	N	-23	20	-	2	78	5
0.2	G	-8	20	5	2	54	2
0.2	M	-8	20	5	2	19	5
0.2	N	-23	20	-	1	67	6
0.5	G	-8	30	2	2	60	2

1.0	G	-8	30	1	2	50	2
1.0	G	-23	30	1	2	47	2
1.0	G	-23	50	1	2	80	2
1.0	G	-23	50	5	1	70	3

Table 2. Simulations of EoR signature detection.

Notes: G = "Galaxy cal" + model of LNA

M = model of LNA + Antenna measurements

N = no model or "Galaxy cal"

Antenna refl. (%) = antenna reflection coefficient

swid(MHz) = span of least squares fit

Cal(%) = percentage perturbation of LNA model

noise(mK) = added noise in 1 MHz bandwidth at 120 MHz

prob(%) = percentage of detections

poly = degree of polynomial used in fit

The antenna data from EZNEC was taken from 80-160 MHz for a relatively poor version of the fourpoint antenna. The maximum reflection coefficient was -8 dB. A much better antenna was "extrapolated" from the first to reach a maximum reflection of -23 dB. This performance is probably the best that can be achieved over a 2:1 frequency range.

Comments of the results

While the combinations of "Galactic and LNA model" calibration is promising it is clear that it will require high model accuracy to reach to delta_z approaching one. Memo#61 shows that the order of the polynomial needs to be low to avoid "soaking-up" the EoR signature in the polynomial and this memo shows that reducing the systematics so that an order polynomial can be used requires accurate antenna impedance measurements combined with a very accurate LNA model.

If "Galactic calibration" is not used the antenna response $f(\nu)$ can be modeled in addition to modeling the $T_{LNA}(\nu)$. In this case the sky noise spectrum has to be assumed. Initial simulations show this method requires even higher accuracy in the LNA model and increased accuracy of the antenna impedance measurement. In all the simulations there is a trade-off between calibration accuracy and antenna match so that a better antenna match generally requires a more relaxed accuracy in the LNA model. In the simulations a 50 ohm impedance was assumed but equivalent or better results might be obtained with 100 ohms provided the 3-position switch reference is also 100 ohms.

In summary the technique of allowing a polynomial to "soak-up" the systematic which result from the lack of a perfect knowledge of the antenna and LNA limit the range of delta_z that can be detected to about 10% of the search bandwidth or 0.4 at 20 MHz and 1.0 at 50 MHz respectively. Beyond these limits the theoretical signal to noise ratio of a detection declines extremely rapidly so that in practice longer integrations may not help.