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To: EDGES Group

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Subject: Simulation of limits imposed by frequency structure in antenna beam

The method of extracting the EOR signature from the ratio of spectra taken when the Galaxy is up to when it is down has been studied in memos 48, 55, and 145. Here is a look at the method again from a simpler perspective.

If we assume linearity

$$Sd = h + g\,Gd \tag{1}$$

$$Su = h + g\,Gu \tag{2}$$

Where Sd and Su are the observed spectra for the Galaxy down and up respectively. h is the global spectrum containing an EoR signature present in both cases. Gd and Gu are the "true" spectra of the "foreground." g is a gain factor spectrum which would be unity if the spectrometer were perfectly calibrated.

Solving equations (1) and (2) for h and eliminating g

$$h = \left(Sd - Su\left(Gd/Gu\right)\right)\Big/\left(1 - \left(Gd/Gu\right)\right)$$

In order to test the effectiveness of the method Gd and Gu can be calculated by convolving the antenna beam with the 408 MHz sky map assuming a fixed spectral index of -2.5 for the entire sky while Sd and Su can be calculated by convolving the antenna beam with a sky map that includes the effects of changes in spectral index with galactic latitude (see memo #8), curvature in the spectral index (from Angelica de Oliveira-Costa's global sky model) and nominal values for the ionosphere at night.

Table 1 gives the values of expected SNR as a function of the EoR signature width for an amplitude of 20 mK centered at 150 MHz. The "noise" is the result of the systematics which appears as a false EoR signature amplitude following subtraction of the best fit polynomial of 6 terms. The row labelled std.dev is the factor by which the effect of random noise effects the EoR estimate. This comes from the square root of covariance for the EoR estimate. Decreasing the number of terms in the polynomial from 6 to 5 decreases the value at 40 MHz from 25 to 6.2 but severely degrades the SNR, used as an indicator of the detectability of EoR owing to the dominance of systematics.

	EoR full width (MHz)							# terms
	10	15	20	25	30	35	40	6
Fourpoint	50	34	22	14	9	5	3	6
Blade	180	128	78	47	28	16	9	6
Dipole	360	225	128	66	39	22	12	6
Std.dev	1.5	2.1	3.1	5	9	15	25	6
Fourpoint								5
Blade	5.4	4.2	3.3					5
Dipole	5.1	4.0	3.1					5
Std.dev	1.3	1.5	1.9	2.5	3.3	4.5	6.2	5

Table 1. Simulated EoR signature detection SNR vs antenna type and EoR width. Assume Gaussian with 20 mK peak amplitude at 150 MHz. Results are given for polynomial fits using 6 and 5 terms. Frequency range 100 to 190 MHz.

While the simulation indicates the detectability of EoR signature when uncertainty in the foreground dominates another test is needed to evaluate the effects of errors in the EM simulation of the beam. In this case Gu and Gd can be derived from a different beam model. Table 2 shows the results for 2 cases. In the first case Gu and Gd are derived using the theoretical beam of a ½ wave dipole at 150 MHz ¼ wave above the ground plane. In the second case Gu are derived from the same antenna offset by 10 degrees in azimuth.

	EoR full width (MHz)							Case
	10	15	20	25	30	35	40	
Fourpoint	14	10	6	4				A
Blade	46	31	20	12	7	4	3	A
Dipole	85	62	43	27	17	10	6	A
Fourpoint	30	21	14	9	5	3		B
Blade	150	106	67	42	26	15	9	B
Dipole	72	54	37	24	14	8	5	B

Table 2. Simulated EoR signature detection SNR

In case "A" Gu and Gd from a theoretical dipole. In case "B" the same antenna is used as for Su and Sd offset by 10 degrees in azimuth. All simulations were run for the latitude of -26 degrees.

Conclusion

These simulations show that under the following assumptions:

1] The radiometric spectrometer is perfectly stable.

2] The radiometric spectrometer is perfectly linear.

3] The contribution to the spectrum from everything except the sky is smooth and would allow easy detection of the EoR after removal of a polynomial of no more than 5 terms.

4] The antenna beam model from FEKO is reasonably accurate.

An EoR signature in the high band of 20 mK and width less than 35 MHz should be detectable using the Fourpoint antenna. The "blade" antenna is significantly better than the Fourpoint and should allow EoR detection up to a width of 40 MHz. These results are applicable to the lowband by dividing the EoR width by 2 and increasing the EoR signature by $2^{2.25}$ = 113 mK