To: EDGES Group  
From: Alan E.E. Rogers  
Subject: Optical depth calculation  

Using the expression

$$\tau = \left[ \frac{3c^3 A h}{(32\pi f^2 k T_s)} \right] N(z) / \Delta v_r(z)$$

Where  
- \( c = \) velocity of light = \( 3 \times 10^8 \) m/s  
- \( A = \) Einstein\( A = 2.85 \times 10^{-15} \) s\(^{-1}\) 
- \( h = \) Planck\’s constant = \( 6.626 \times 10^{-34} \) m\(^2\)kg/s \( f = \) frequency = \( 1.42 \times 10^{29} \) Hz \( k = \) Boltzmann\’s constant = \( 1.38 \times 10^{-23} \) m\(^2\)kg s\(^{-2}\)/K \( T_s = \) spin temperature = 20 K \( N(z) = \) number H atoms/m\(^3\) = 4% of total mass = 0.21 \((1 + z)^3\) m\(^{-3}\) \( \Delta v_r(z) = \) velocity spread/m = \( 69 \times 10^3 / 3.086 \times 10^{22} (1 + z)^{1/2} \) s\(^{-1}\) 

= 0.02 for \( z = \) redshift = 16


I obtained the constants from [www.astro.caltech.edu/~george/ay127/Ay127.contents.pdf](http://www.astro.caltech.edu/~george/ay127/Ay127.contents.pdf) and got the same results using cgs and MKS units. The main point of this is to show that under the assumption of a smooth Universe (Liddle 2015) and \( T_s = 20 \) k the hydrogen optical depth is much less than one. I have been unable to find much about the effects of clumps in the hydrogen clouds. Clumping without changes in \( T_s \) probably only reduces the depth of the absorption signature via regions of high opacity which saturate the absorption.

Based on the contributions to the spin temperature in Table 3 of Field (1958) the higher densities expected in clumps will tend to pull \( T_s \) closer to the kinetic temperature due to the increased rates of collisions.
The peak absorption is given by

\[ (1 - e^{-z}) \left( \frac{T_i - T_{\text{CMB}}}{1 + z} \right) \]

where \( T_{\text{CMB}} = 2.75 (1 + z) \)

which could have a value up to about 0.4 K if the spin temperature is coupled to a kinetic temperature as low as 3K as in a model of Prober et al. (2015) (Ap.J. 809(1), 62).


