EDGES MEMO # 221

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To: EDGES Group

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Subject: Optical depth calculation

Using the expression

$$\tau = \left[3c^3 Ah \Big/ \left(32\pi f^2 kT_s\right)\right] N(z) \Big/ \Delta vr(z)$$

Where c = velocity of light – 3×10^8 m/s

A = Einstein A $= 2.85\times10^{-15} s^{-1}$

h = Planck's constant = $6.626\times10^{-34}\ m^2 kg / s$

f = frequency = 1.42×10^9 Hz

k = Boltzmann's constant = $1.38\times10^{-23}\ m^2 kg\ s^{-2} / K$

Ts = spin temperature = 20 K

N(z) = number H atoms/m^3 = 4% of total mass = $0.21\ (1+z)^3\ m^{-3}$

$\Delta vr(z) =$ velocity spread/m $= 69\times10^3 / 3.086\times10^{22} (1+z)^{3/2}\ s^{-1}$

$= 0.02$ for z = redshift = 16

From Furlanelto et al (2006) and Ciardi et al (2010).

I obtained the constants from www.astro.caltech.edu/~george/ay127/Ay127.contents.pdf and got the same results using cgs and MKS units. The main point of this is to show that under the assumption of a smooth Universe (Liddle 2015) and Ts=20 k the hydrogen optical depth is much less than one. I have been unable to find much about the effects of clumps in the hydrogen clouds. Clumping without changes in Ts probably only reduces the depth of the absorption signature via regions of high opacity which saturate the absorption.

Based on the contributions to the spin temperature in Table 3 of Field (1958) the higher densities expected in clumps will tend to pull Ts closer to the kinetic temperature due to the increased rates of collisions.

The peak absorption is given by

$$\left(1-e^{-\tau}\right)\left(T_s - T_{CMB}\right)/\left(1+z\right)$$

where $T_{CMB} = 2.75\ (1 + z)$

which could have a value up to about 0.4 K if the spin temperature is coupled to a kinetic temperature as low as 3K as in a model of Prober et al. (2015) (Ap.J. 809(1), 62).

Furlanetto, S. R., S. P. Oh, F. H. Briggs (2006), Cosmology at low frequencies: The 21 cm transition and the high-redshift Universe, PhR, 433, 181-301, doi:10.1016/j.physrep.2006.08.002.

Ciardi, B., R. Salvaterra, T. Di Matteo (2010), Lyα versus X-ray heating in the high-z intergalactic medium, MNRAS, 401, 2635-2640, doi:10.1111/j.1365-2966.2009.15843.x.

Field, G.B (1958) Excitation of the Hydrogen 21-CM line, Proceedings of the IRE, 240.

Liddle, A. (2015) An Introduction to modern cosmology, wiley, SBN: 978-1-118-50214-3.

Loeb, A. (2010) How did the First Stars and Galaxies Form? Princeton University Press