A Proposed Solar Wind-Magnetosphere-Ionosphere Coupling Model

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Penetration Electric Field

Outline

• Introduction
• The Ionospheric Model
• The Magnetospheric Models
• Joining the Three Models
• Determine $E_{eq}(t)$ vs. $E_{sw}$ (solar wind $E$)
• Conclusions
Penetration Electric Field

Solar Wind: Hill-Siscoe L1 -> Transpolar Potential (Theory)
           Weimer Experimental Transpolar Potential

Ionosphere: Nopper-Carovillano Model

  two-cell polar convection cells
  electrojets
  global ionospheric electric fields


Goal: Time history of penetration E-field at Equatorial Latitudes during the magnetic storm of 6-7 April 2000.
# Strengths and Weaknesses

## Comparing the Models

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<th>Strength</th>
<th>Weakness</th>
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<td><strong>N-C</strong></td>
<td>Global Ion. Pot.</td>
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<td><strong>Hill-Siscoe</strong></td>
<td>$SW(L1) \rightarrow Φ_{pc}$</td>
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<td>Hi-Lat. Pot. IMF Effects</td>
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<tr>
<td></td>
<td>No IMF-SW, $Φ_{pc}$</td>
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<tr>
<td></td>
<td>Undefined Ion.</td>
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<td></td>
<td>Undefined Circuit Elements, $Φ_{pc}$</td>
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<tr>
<td></td>
<td>No Low-Lat. Ion. Pot.</td>
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Penetration of Electric Field

Nopper – Carovillano Model (N-C)

• The Nopper-Carovillano (N-C) model defines the global distribution of ionospheric electric fields and currents as a function of Region I and Region II auroral field-aligned currents.
  – FAC’s are the inputs to the model.
• The ionosphere is modeled as a single-layer
  – Noon-midnight asymmetry in conductivity
  – Annular region of enhanced conductivity at auroral latitudes.
• In the present work we treat both hemispheres
  – Symmetry between the two polar regions is not required.
• The N-C Model predicts the two-cell convection pattern in the polar region.

Penetration of Electric Field

Nopper – Carovillano (N-C) Model

• Start with 3-D Ionospheric Current

\[ J = \sum_o (E \cdot b) b + \sum_p [E - (E \cdot b) b] + \sum_H b \times E \]

• \( b = \) unit vector along \( B \); IMPOSE \( J_r \) (vertical) = 0

• The restriction of the 3-D current to a 2-D spherical shell leads to a radial polarization electric field \( E_r (E_\theta, E_\phi) \), which also depends on the dip angle \( (I) \) of the dipole magnetic field and the ionospheric conductivities.
Global Pedersen Conductance

\[ \Sigma p(\theta, \phi) = \beta(F10.7) \exp\{-[\cos^{-1}(\sin(\theta)\cos(\phi))]^2/1.804\} \]

+ Gaussian with amplitude \( \Sigma_A \) centered near J1

**Subsolar:**

\[ \Sigma p[\theta=90^\circ, \phi = 0^\circ \text{ (MLT= 12:00)}] = \beta(F10.7=177) = 18.7 \text{ S} \]
N-C Model

\[ J_\theta = \Sigma_{\theta\theta} E_\theta + \Sigma_{\theta\phi} E_\phi \]

\[ J_\phi = -\Sigma_{\theta\phi} E_\theta + \Sigma_{\phi\phi} E_\phi \]

\[ \Sigma_{\theta\theta} = \Sigma o \Sigma p / D \]

\[ \Sigma_{\theta\phi} = \Sigma o \Sigma H \sin(I) / D \]

\[ \Sigma_{\phi\phi} = \frac{[\Sigma o \Sigma p \sin^2(I) + (\Sigma p^2 + \Sigma H^2) \cos^2(I)]}{D} \]

\[ D = \Sigma o \sin^2(I) + \Sigma p \cos^2(I) \]

\[ I = \text{DIP ANGLE} \]

\[ \sin(I) = 2\cos(\theta) / \sqrt{(1 + 3 \cos^2(\theta))} \]
\[ \nabla \cdot J_i = - j_r \text{ (input)} \]

\[ \nabla \cdot [ \Sigma \cdot \nabla \Phi ] = j_{||} \sin (l) \]
N-C  $J_1 = 1 \text{ MA}$  $J_2 = 0 \text{ MA}$
Polar Cap Potential (kV)

\[ \Phi_{pc}(1) = 107 \text{ kV} \]
\[ \Phi_{pc}(2) = 153 \text{ kV} \]
\[ \Phi_{pc}(W) = 84 \text{ kV} \]

- J1 = 1 MA  J2 = 0.5 MA
- J1 = 1 MA  J2 = 0.0 MA
- Weimer(1995)  Fig. 9
N-C Model

ELECTRIC FIELD  J1 = 1 MA  J2 = 0 MA  $\Sigma_A = 3$ S

20 mV/m

$\lambda_m$
N-C Model

\[ J_w = 1.1 \text{ MA} \quad J_E = 1.4 \text{ MA} \]

J ionosphere   \[ J_1 = 1 \text{ MA} \quad J_2 = 0 \text{ MA} \]

200 mA/m
Penetration Electric Field
J1 = 1 MA, J2 = 0 MA – N-C Model
Penetration Electric Field
(Cond. Gradients along terminator) – J-G Model

\[ J_1 = 1 \text{ MA}, \ J_2 = 0 \text{ MA} \quad \text{Eq}q_1 \sim 1 \text{ mV/m} \quad \text{Eq}q_2 \sim -2 \text{ mV/m} \]
Result #1

- \[ E_{eq}(t) = J_1 E_{eq1} + J_2 E_{eq2} \]

- \[ E_{eq1} \sim 1 \text{ mV/m} \quad E_{eq2} \sim -2 \text{ mV/m} \]
Φρс N-C

Polar Cap Potential (kV)

Φρс (1) = 107 kV
Φρс (2) = 153 kV
Φρс (W) = 84 kV

J1 = 1 MA    J2 = 0.5 MA
J1 = 1 MA    J2 = 0.0 MA
Weimer(1995)    Fig. 9
Hill – Siscoe – Ober Model

• Reconnection at the Dayside Magnetopause is the Source of the Transpolar Potential $\Phi_{PC}(H-S)$
  
  $\Phi_{PC}(H-S)$ Drives the Region I Current ($J_1$).

• Magnetic Feedback from $J_1$ to the Dayside Magnetopause Causes $\Phi_{PC}$ To Saturate.

• $\Phi_{PC} = \frac{30P_{sw}^{1/2} + 57.6 E_{sw} P_{sw}^{1/3} \sin^2 (\chi)}{0.0187 \xi \Sigma P_{sw}^{1/6} + 0.036* \xi \Sigma P E_{sw} \sin^2 (\chi) + P_{sw}^{1/2}}$

$E_{sw}, P_{sw}$ = solar wind electric field and pressure

$J_1 = \Phi_{PC} \Sigma P \xi$  \hspace{1cm} $\xi > 1$  \hspace{1cm} $\Sigma P = 0.37 \text{ SQRT } (F10.7)$

Efficiency of ionosphere to close currents
Example of Transpolar Potential Saturation

Hill – Siscoe Model

Example of Polar Cap Potential $\Phi_{pc}(H-S)$ From Magnetopause Reconnection

$P_{sw} = 4.47 \quad F_{10.7} = 80.5 \quad \Sigma p^p = 6.9 \, S \quad \xi = 2.9$

Note saturation at higher values of the solar wind electric field $E_{sw}$
ACE L1 Data 6 - 7 April 2000
Shifted in Time by Vx

1. \( P_{SW} \) (nPa)

2. \( E_{SW} \) (mV/m)

3. \( D_S \) (nT)

4. \( B_T \) (nT)
Weimer and H-S-O

Weimer and H-S-O  Φpc - 6-7 April 2000 Magnetic Storm

<table>
<thead>
<tr>
<th>UT</th>
<th>Φpc (kV)</th>
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<tbody>
<tr>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>75</td>
</tr>
<tr>
<td>12</td>
<td>100</td>
</tr>
<tr>
<td>18</td>
<td>125</td>
</tr>
<tr>
<td>24</td>
<td>150</td>
</tr>
<tr>
<td>30</td>
<td>175</td>
</tr>
<tr>
<td>36</td>
<td>200</td>
</tr>
<tr>
<td>42</td>
<td>225</td>
</tr>
<tr>
<td>48</td>
<td>250</td>
</tr>
</tbody>
</table>

- **H-S-O 0 S**: Solid line
- **H-S-O 3 S**: Dashed line
- **Weimer**: Blue diamonds

Unclassified: Distribution Unlimited
Result #2

• The Hill-Siscoe-Ober Model gives a good estimate of the transpolar potential $\Phi_{pc}$ during this magnetic storm.

• We now have $E_{eq1}(J1=1 \text{ MA})$, $E_{eq2}(J2=1 \text{ MA})$, and a driver for $J1$.

What about $J2$?
S-RC Siscoe [1982]

Diagram showing two current loops with labeled regions and resistances.
Dst and $\Phi pc$

Year 2000

$\Phi pc(\Sigma A=3S)$

$\text{Dst}$

Julian Day
S-RC  Siscoe J1, J2 Coupling
S-RC

- I  \( J_1 = \frac{\Phi p_c}{R_p} + J_2' \)
- II  \( (J_2' - J_2)R_s + 2R_A J_2' = \Phi p_c \)
- III  \( L dJ_2/dt + (J_2 - J_2') R_s = 0 \)

\[ J_2 = \frac{\Phi p_c}{(2R_A)}(1 - \exp(-t/\tau)) \]
\[ \tau = L(R_s + 2R_A)/(2R_A R_s) \]

\[ J_1 = \Phi p_c\left[\frac{1}{R_p} + \frac{1}{(R_s + 2R_A)} + \ldots + \frac{R_s}{(2R_A (R_s + 2R_A))} (1 - \exp(-t/\tau)) \right] \]
What About L?

ELECTRICAL ENERGY -> ION GYRO-ENERGY

- \(\frac{dW}{dt} = qV_d \cdot E_y\)

\(V_d = B\)-GRADIENT DRIFT

HINES [1963]

INDUCTANCE IS THE ELECTRICAL ANALOG TO INERTIA (MASS)
S-RC Siscoe [1982]
What About L?

- \( R_p = 0.17 \ \Omega \quad R_A = 0.018 \ \Omega \quad R_s = 0.11 \ \Omega \)

- **Dessler-Parker-Scopke Relation for Ring Current:**
  
  TOTAL RING CURRENT ENERGY = \( 2.68 \times 10^{13} \) Dst(nT) J

- \( \frac{1}{2} L J_2^2 = 2.7 \times 10^{13} \) Dst

- Assume: \( J_2 = \Phi p c / (2RA) \) (max value) when Dst at min

- \( \tau = 1.12 \times 10^{14} \) Dst \( (R_s + 2RA)RA / (R_s \Phi p c^2) \) s

- \( \tau = 5.5 \) h \( \quad L = 545 \) H

\( \tau \) is not the shielding time, it is the rise time of J2
Penetration Electric Field
(Putting it Together)

• \( E_{eq}(t) = \Phi_{pc}(t) \left( \frac{1}{Rp} + \frac{1}{Rs + 2R_A} \right) + \frac{Rs}{(2R_A (Rs + 2R_A))} \ldots \times \left(1 - \exp\left(-\frac{t}{\tau}\right)\right) E_{eq1} + \Phi_{pc}/(2R_A)(1-\exp(-t/\tau)) E_{eq2} \)

• \( E_{eq1} = 1 \text{ mV/m, } E_{eq2} = -2 \text{ mV/m } \text{ from N-C} \)
• \( Rp = 0.17 \Omega \quad R_A = 0.018 \Omega \quad Rs = 0.11 \Omega \)
J1 and J2 6 - 7 April 2000 Magnetic Storm
Penetration Electric Field

Eeq(mV/m)  6 - 7 April 2000  Magnetic Storm

![Graph showing Eeq(mV/m) over time with various lines and markers for different conditions.]

- Rp = 0.17, RA = 0.018, Rs = 0.11
- Rp = 0.17, RA = 0.036, Rs = 0.11
- Rp = 0.17, RA = 0.036, Rs = 0.05
- Bubbles

50 m/s

16:00 18:00 20:00 22:00 24:00 02:00 04:00 06:00

UT
Conclusions

• The three models considered can be joined in a manner such that the penetration electric field at equatorial latitudes can be related to the solarwind as measured at L1.

• Within the uncertainties of the model we believe that the first set of bubbles are caused by penetration electric fields.

• This model should be useful for C/NOFS studies