Resolution Metrics for Auroral Multipoint Measurements with Satellite Swarms

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Abstract

With the growing accessibility of low-cost satellite measurements, Boston University set out to create a space-based swarm called ANDESITE to do multipoint magnetometer measurements of the auroral current systems. Here, we show a model of the system and a rigorous, physically consistent, method to model its capability. With this methodology, any formation geometry can be evaluated for its ability to resolve postulated plasma wave phenomena that are superimposed onto the field aligned currents in the aurora. These investigations inform the current design and future mission concepts that could help decipher energy transport in the magnetospheric-ionospheric coupling that occurs at high latitude.

I. Doing More with Small Satellites

Problem:
The fine-scale structure of the near-earth electromagnetic environment is not well understood due to lack of *in situ* measurements

$$\nabla \times \left(\vec{B}_{\text{Earth}} + \delta\vec{B} \right) = \mu_0 \vec{J}$$

Possible Solution:
- Ideal problem to demonstrate small distributed space-based sensors
- CubeSats offer a platform for development and deployment of cheaper spacecraft and are ideal for such scientific measurements
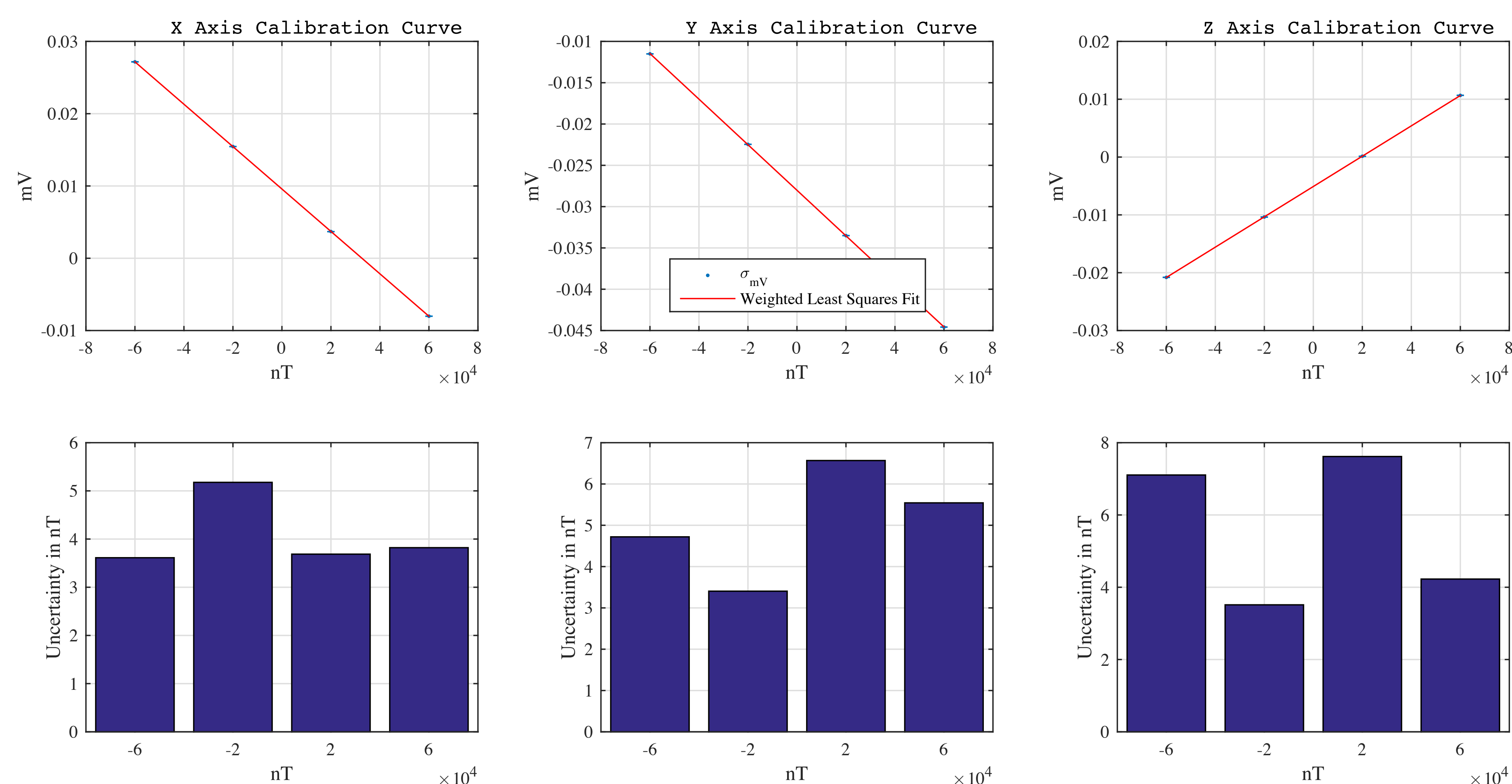
II. The Spacecraft Sensor
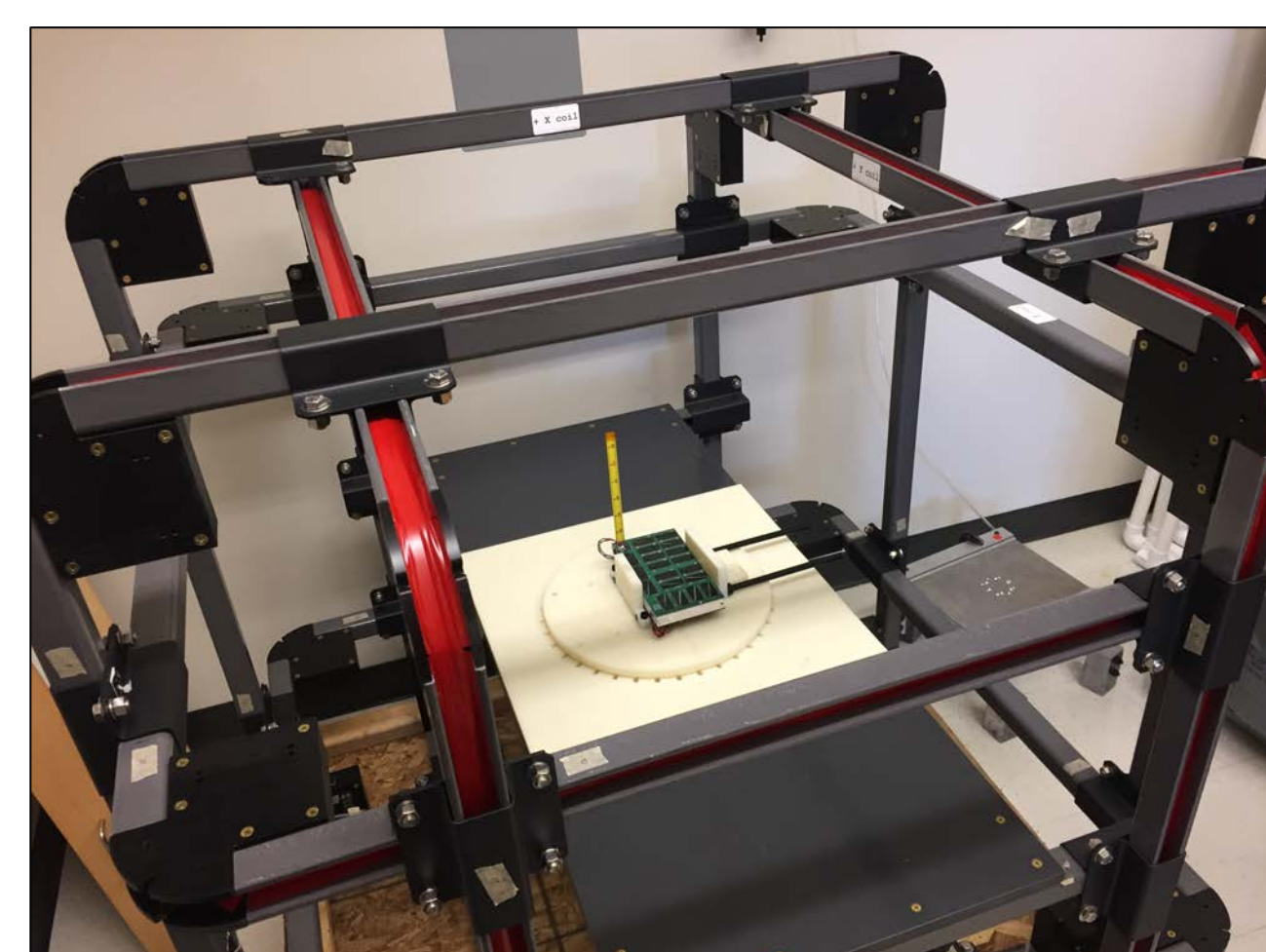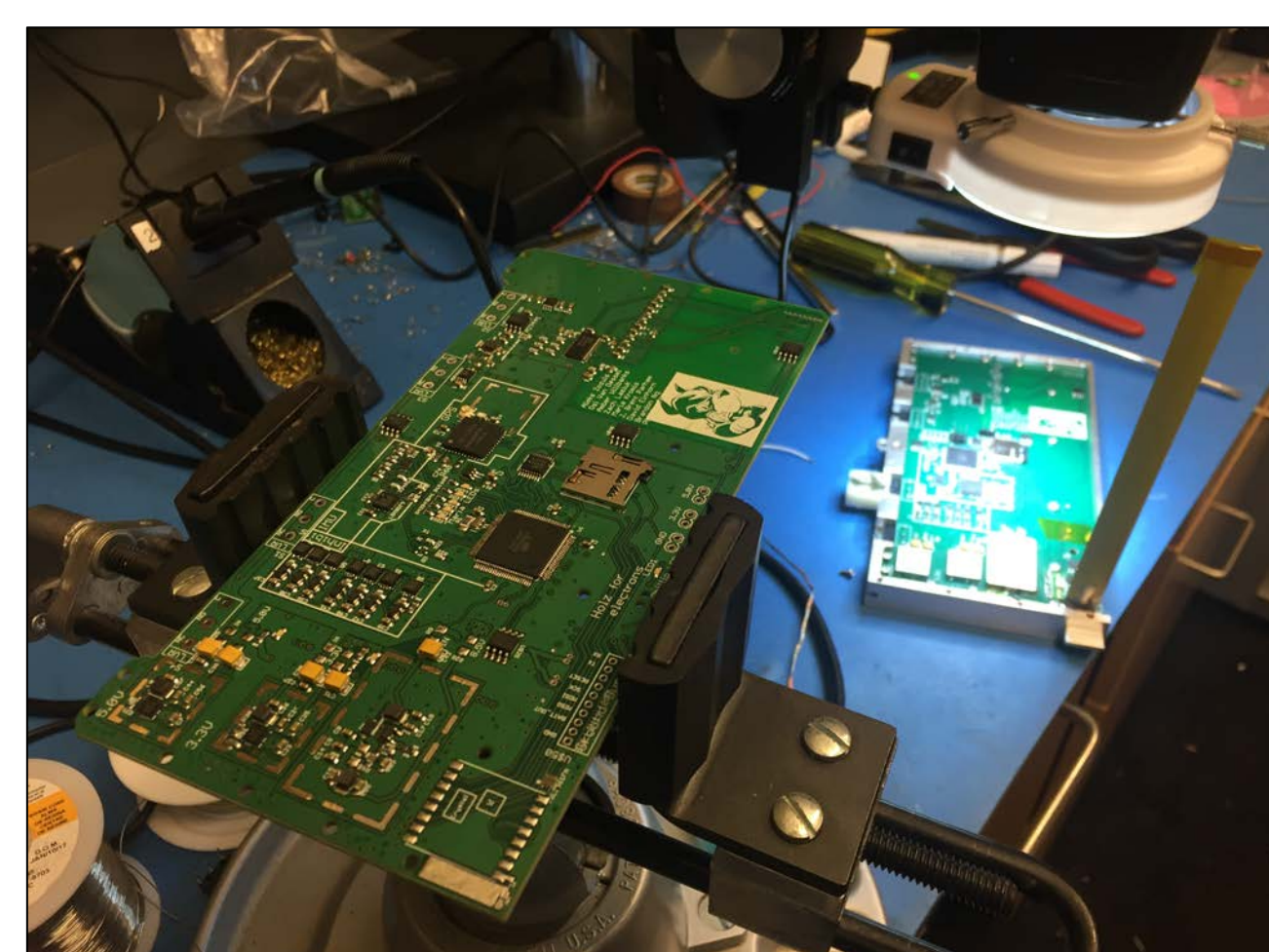
The **image to the far left** shows the finalized printed circuit board design of sensor node (the picosatellite). The magnetometer payload is the top right corner of the image.

The **right photograph** shows an engineering model of the same satellite in the Helmholz cage used for the below calibration.
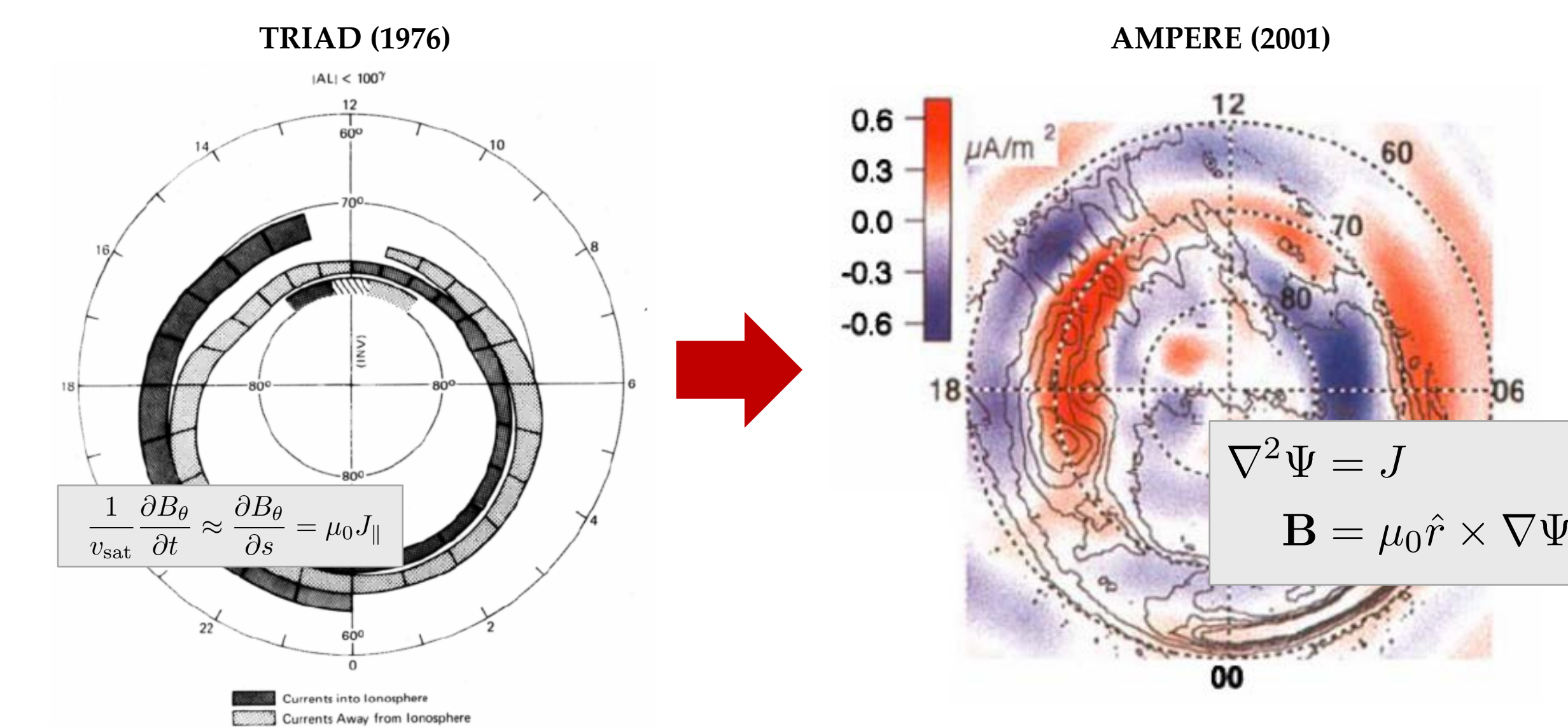

As seen above, although we are using a relatively inexpensive magnetometer and housing it inside the spacecraft, the standard deviation of the noise stays within 10 nT on each axis throughout a dynamic range of 120,000 nT. Any offsets due to non-orthogonality, inherent structural magnetic moments, and other hardware dependent causes are calibrated out with extensive probing of the the spacecraft at multiple angles of attack with the Helmholz cage.

This achievement is due to careful design considerations and electromagnetic shielding placed within the spacecraft. Often the easiest way to check the feasibility of a sensor concept is to build and test hardware and not rely on preconceived notions.

III. Metrics of Resolution

ANDESITE follows in a tradition of ever increasing sensor resolution (think TRIAD's coarse "up" or "down" compared to AMPERE's finer global picture seen today). But what does that mean quantitatively?

Below, an analytical harmonic model of the electro-magnetic field and current system is used to gauge the ability of ANDESITE. Imagine the spacecraft swarm as a 2D spatial filter reconstructing the field from discrete measurements.

$$\frac{1}{v_{\text{sat}}}\frac{\partial B_\theta}{\partial t} \approx \frac{\partial B_\theta}{\partial s} = \mu_0 J_\parallel$$

$$\nabla^2 \Psi = J$$
$$\mathbf{B} = \mu_0 \hat{r} \times \nabla \Psi$$

Discretized Field

This represents the discretized view of the current system as seen by the swarm. Each triplet of data samples form a triangle that can estimate a current density

Differential Form
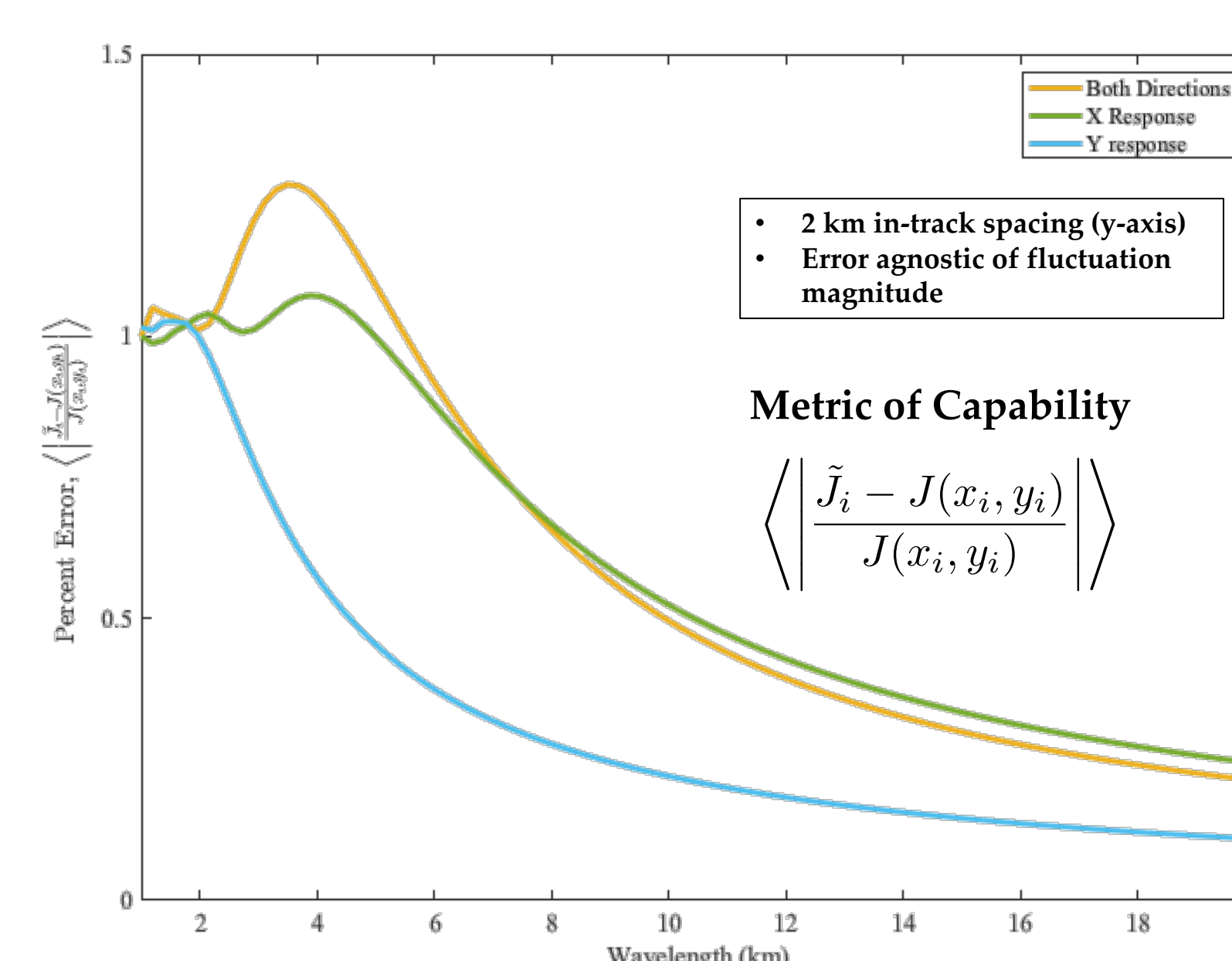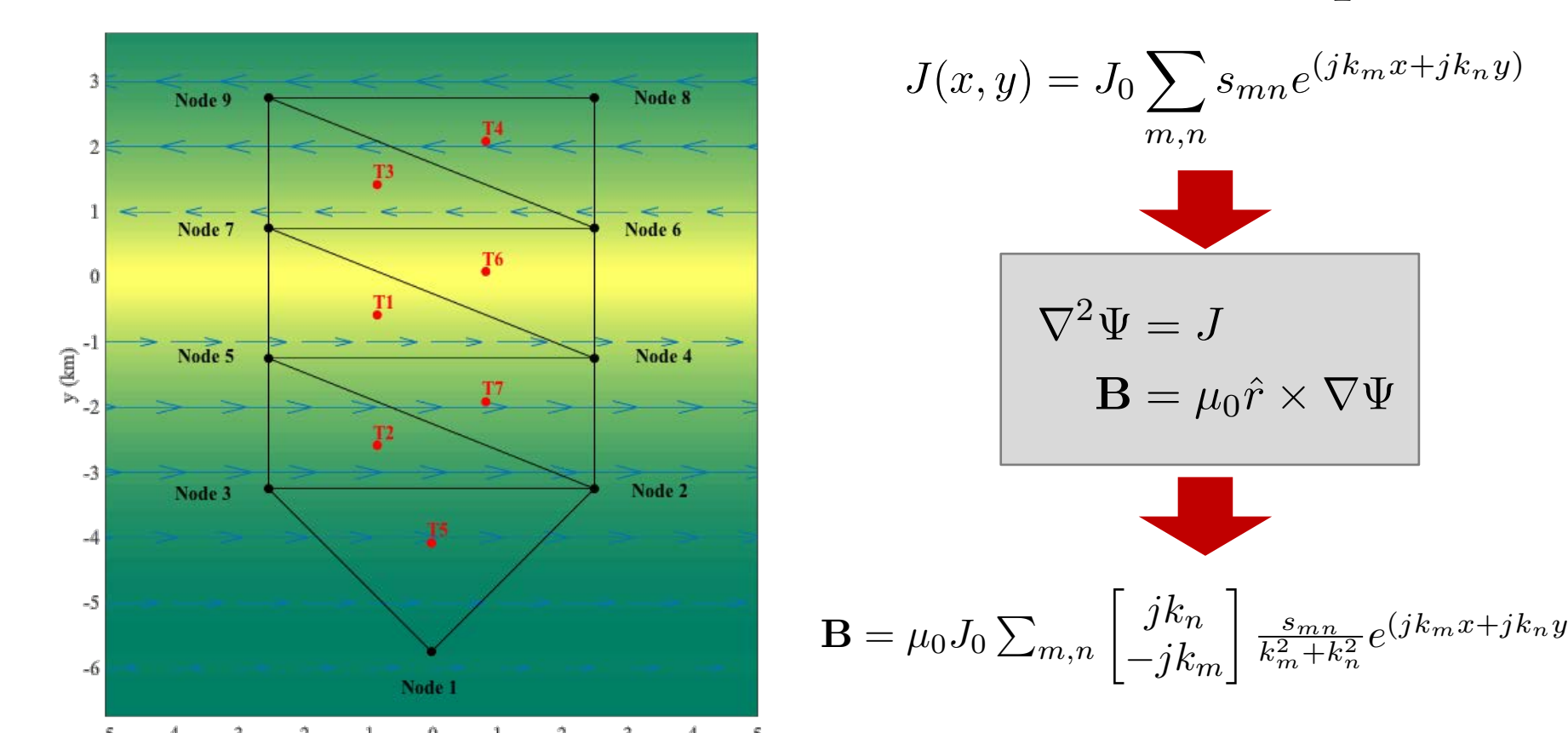$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

Integral Form
$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int\int \mathbf{J} \cdot d\mathbf{A}$$

$$\left(\frac{\mathbf{B}_1 + \mathbf{B}_2}{2}\right) \cdot \mathbf{r}_{12} + \left(\frac{\mathbf{B}_2 + \mathbf{B}_3}{2}\right) \cdot \mathbf{r}_{23} + \left(\frac{\mathbf{B}_3 + \mathbf{B}_1}{2}\right) \cdot \mathbf{r}_{31} = -\mu_0 \vec{J}\frac{|\mathbf{r}_1 2 \times \mathbf{r}_1 3|}{2}$$

Analytical Reference and Sampling

Below is the harmonic expansion of the magnetic field (vectors in the image) and it's corresponding current density (gradient color representing the magnitude). The image also shows a hypothetical ANDESITE swarm and the nodes at which data is acquired.

$$J(x,y) = J_0 \sum_{m,n} s_{mn} e^{(jk_m x + jk_n y)}$$

$$\nabla^2 \Psi = J$$
$$\mathbf{B} = \mu_0 \hat{r} \times \nabla \Psi$$

$$\mathbf{B} = \mu_0 J_0 \sum_{m,n} \begin{bmatrix} jk_n \\ -jk_m \end{bmatrix} \frac{s_{mn}}{k_m^2 + k_n^2} e^{(jk_m x + jk_n y)}$$

- 2 km in-track spacing (y-axis)
- Error agnostic of fluctuation magnitude

Metric of Capability
$$\left\langle \left| \frac{\tilde{J}_i - J(x_i, y_i)}{J(x_i, y_i)} \right| \right\rangle$$

By the framework developed above we can calculate a frequency response of swarm (**plot to the left**). These non-dimensional curves in the metric of capability allow us understand the relative sensitivity to different spatial scales.

Varying the deployed distances and assuming a relevant current density, we can then see the trade off in discrete spatial sampling of the field and required magnetic field sensitivity (**plot to the right**).
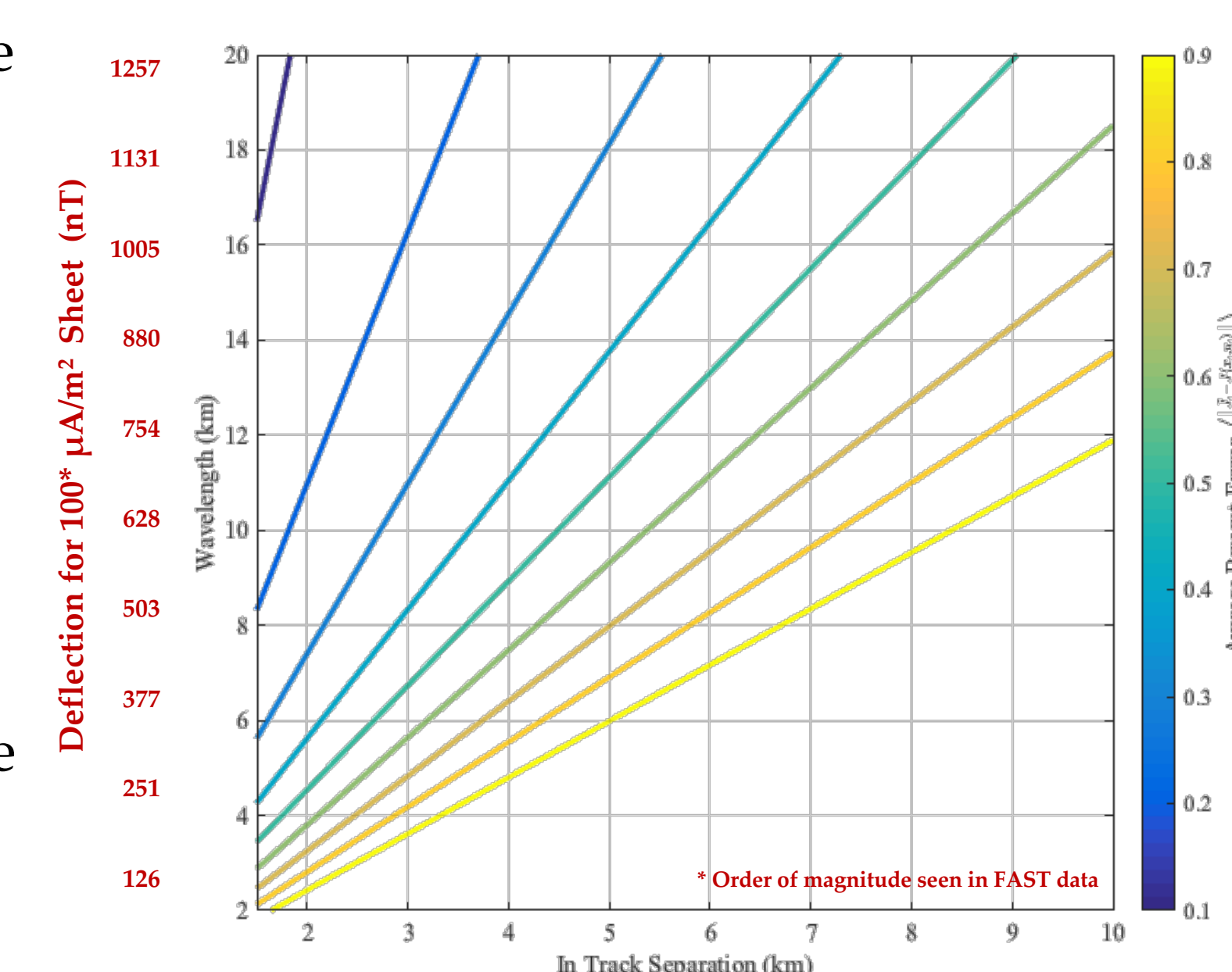
IV. The Real Test—Launch.

ANDESITE was designed to push the limits of spatial resolution and here we developed a rigorous methodology to examine the performance of satellite swarms like it, ultimately showing that we are mostly limited by the spatial separation of our sensor measurements and not the magnetometer sensitivity. We did still prove the capability of the system to resolve kilometer scale phenomena that include the domain of postulated in situ Alfven waves. Future work along this direction will be to better understand the role of measurement uncertainty with the approach defined herein. Through understanding the uncertainty we can then better inform the magnetometer design for future mission and rigorously examine the one chosen for ANDESITE.

However, the real test is when the sensor is launched. No amount of ground-based testing or modeling can accurately characterize a system and predict what we may measure once on orbit.
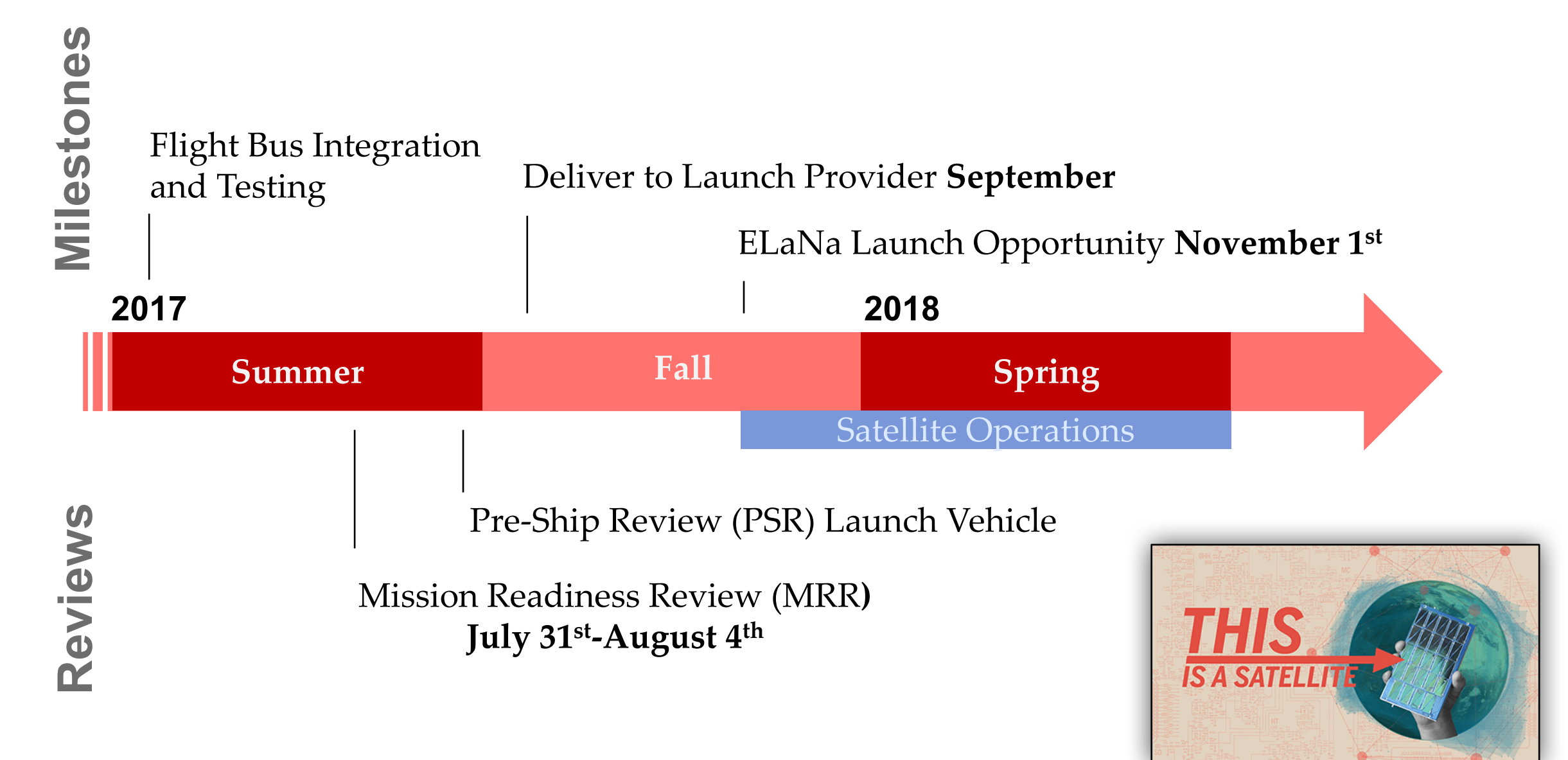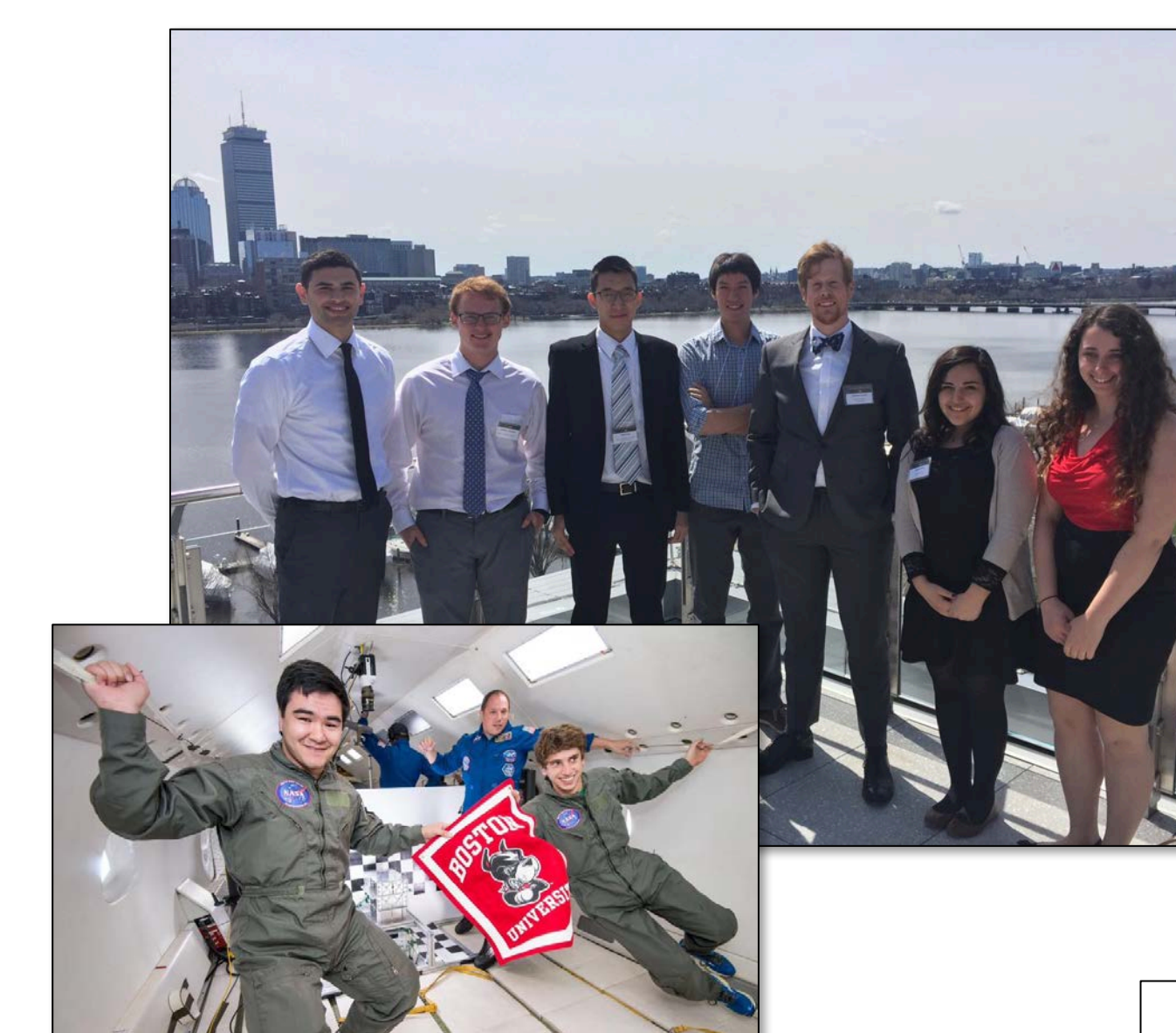
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Relevant References for Science Mission

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