DEUTERIUM ARRAY MEMO #003
LOFAR MEMO #001

# MASSACHUSETTS INSTITUTE OF TECHNOLOGY
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To: Deuterium Array Group

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Subject: The sensitivity of a sky noise dominated array

Introduction:

At frequencies below about 500 MHz the sky noise dominates the system temperature as the amplifier noise from a pseudomorphic high electron mobility field effect transistor (PHEMT) drop to below 10 K at frequencies below 1 GHz. The sky noise below 500 MHz varies by more than a factor of 10 from regions near the Galactic plan to regions well away from the Galactic plane. In this regime the signal to noise ratio of the array depends on the collecting area and the sidelobe level. For example a compact array can reject the noise of the Galactic plane when looking out of the plane and achieve a system temperature equal to the sky background away from the plane. A sparse array cannot reject the noise simultaneously from all points in the Galactic plane and will only achieve a noise temperature system equal to the sky background averaged over the response of an individual element. At 150 MHz the background away from the plane is about 200 K compared with an average background of about 500 K.

Signal to noise calculation

The signal power P from a point source of flux density s assuming each element is correctly phased and the elements are equally weighted with voltage weighting of $N^{-½}$

$$P = \frac{1}{N}\left(\sum_N \left(\frac{SA}{2k}\right)^{1/2}\right)^2 = \left(\frac{S}{2k}\right)NA \quad K$$

where N = number of elements
A = collecting area of each element
k = Boltzmann's constant

The noise power NP is given by

$$NP = \frac{1}{N}\left(\sum_N \left(Tr_n\right)^{1/2} + \left(\frac{1}{4\pi}\right)^{1/2} \sum_M \sum_N \left(G_m T_m \Delta\Omega\right)^{1/2} e^{i\theta_{mn}}\right)^2$$

where $Tr_n$ = the receiver noise for the $n^{th}$ element
$G_m$ = the element directivity in the direction of the $m^{th}$ patch of sky
$T_m$ = sky brightness for $m^{th}$ patch of sky
$\Delta\Omega$ = solid angle of patch
$\theta_{mn}$ = phase of $n^{th}$ element towards $m^{th}$ patch of sky

Since the receiver noise is uncorrelated

$$NP = T_r + \frac{1}{4\pi N}\sum_{M}\sum_{M'}\sum_{N}\sum_{N'} G_m^{1/2} G_{m'}^{1/2} T_m^{1/2} T_{m'}^{1/2} \Delta\Omega e^{i\theta_{mn}} e^{-i\theta_{m'n'}}$$

and since the signals from different sky patches are uncorrelated

$$NP = T_r + \frac{1}{4\pi N}\sum_{M}\sum_{N}\sum_{N'} G_m T_m \Delta\Omega\, e^{i(\theta_{mn}-\theta_{mn'})}$$

If the elements are widely spaced the element phases are uncorrelated in directions except in the direction of the main beam

$$\begin{aligned} NP &= T_r + \frac{1}{4\pi N}\sum_{M>0} N G_m T_m \Delta\Omega + \frac{1}{4\pi N} G_0 T_0 \Delta\Omega N^2 \\ &= T_r + T_{av} + T_{mb} \end{aligned}$$

where $T_{av}$ is the sky brightness averaged over the directivity of an array element.

If the elements are spaced in a regular 2-D array with spacing d the array will have zero response for Nyquist sampling of the sky out to an angle of $\pm\sin^{-1}\left(\frac{\lambda}{2d}\right)$ in all directions except that towards the source.

In the case

$$NP = T_r + T_{mb}$$

where $T_{mb} = \frac{1}{4\pi N} G_0 T_0 \Delta\Omega N^2$

and $\Delta\Omega = \left(\frac{\lambda}{N^{1/2}\, 2d}\right)^2$

so that $NP = T_R + T_0\left(\frac{G_0\lambda^2}{16\pi d^2}\right)$

where $T_o$ is the sky temperature in the direction of the source averaged over the main beam of the array. If we now assume that the antenna elements have a uniform response over a solid angle of $\left(\frac{\lambda}{2d}\right)^2$ and zero response elsewhere

$$G_0 = 16\pi\, d^2 / \lambda^2$$

and $\quad NP = T_R + T_0$

For dipole antennas with a ground plane the antenna spacing d must be less than $\lambda/2$ to sample the entire hemisphere and achieve a array system temperature equal to that of the brightness averaged over the main beam. Larger spacing will result in aliasing lobes in the case of uniform spacing or random sidelobes for random spacing which will increase the system temperature by an amount up to the average sky brightness over the individual antenna beams. In observing the galactic polar regions at 150 MHz a compact array (dipole element spacing $<\lambda/2$) could achieve 200 K system while a sparse array might only achieve a 500 K system. If the elements have a response which cover less than a hemisphere the spacing can be increased to more than a half wavelength.