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To: Deuterium Array Group

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Subject: "Standing Waves" in arrays

Single dish spectroscopy is often limited by "standing waves." These "standing waves" produce periodic ripples in the spectra. The spurious spectral features are the result of several mechanisms. (see for example MM-VLBI memo #7). Arrays are not immune from "standing waves". Mechanisms which may produce significant are:

1] False correlations due to emissions of noise from LNAs.

2] Scattering and cross-coupling of signals from strong sources like the sun.

The spectrum from a beam of the array is given by

$$S(\omega) = \left\langle X(\omega) X^*(\omega) \right\rangle \tag{1}$$

where

$$X(\omega) = \sum_j e^{i\phi_j} \left(n_j + \sum_{k \neq j} n_k c_k p_{jk} \right) \tag{2}$$

where n_j = normalized noise $\left(\left\langle \left| n_j \right|^2 \right\rangle = 1 \right)$ from jth element

ϕ_j = phase shift applied to jth element to form beam

c_j = correlation coefficient of radiated noise with respect to the received signal from jth element

p_{jk} = voltage coupling between jth and kth element

For elements separated by distance d

$$p_{jk} \simeq \frac{e^{i\omega\tau} g_{jk} \lambda}{4\pi d} \tag{3}$$

where $\tau = d / c$ (4)

g_{jk} = jth element gain in the direction of the kth element

If we assume that

$c_j \approx 1$

$|p_{jk}| \approx p$ i.e. similar magnitude for all coupling pairs.

The phase of p_{jk} is random

and $\langle n_j n_k \rangle = 0$ when $j \neq k$

so that:

$$S(\omega) \approx N + \left(N(N-1)\right)^{1/2} p \tag{5}$$

where the square root arises from the assumption of random phases producing a random walk. This shows that the fractional ripple amplitude in the spectrum is approximately equal to the magnitude of the voltage coupling. This is similar to the result obtained for a single dish. For example the magnitude of the ripple due to the reflection of receiver noise from the subreflector back into the feed of a cassegrain antenna is approximately equal to the square root of the path loss for a reflection from the subreflector or about 10^{-3} for the geometry of Haystack.

If we assume that only adjacent antenna elements have a coupling magnitude of p then each element will produce a correlated signal in 8 other elements for a regular grid and

$$S(\omega) \approx N + (8N)^{1/2} p \tag{6}$$

and for large N $S(\omega) \approx N\left(1 + 2\sqrt{2}N^{-1/2}p\right)$ (7)

so that the relative magnitude declines as N increases. In the case of a 5×5 array for each station

$$S(\omega) \approx 25(1 + 0.6p) \tag{8}$$

and we might expect a ripple of about 2K if the cross-coupling is about –10 dB and the correlated part of the radiated noise is 10K. Fortunately the period of the ripple is on the order of 500 MHz because elements are about 24 inches apart. If the array is used as a synthesis instrument coupling between stations would produce baseline ripples of the order of 1 MHz period for stations 100 m apart. However in this case the coupling would be expected to be very much weaker – perhaps as low as –100 dB so that ripples would now be of the order of $5 \times 10^{-5} K$. My tentative conclusion is that radiated noise which correlates with the received noise (often known as the "Weinreb effect" in the case of a single dish) and produces ripple in the spectrum should not be a problem.

At 327 MHz the sun is the strongest source in the sky and can reach 10^8 J although normally around 3×10^5 J. The received noise in each dipole (assuming a gain of 6 dBi) is about 30 K at 3×10^5 J. Some of this noise can be coupled into other elements and produce spurious correlations. However the magnitudes and ripple periods will be similar to the radiated receiver noise and should not be a problem except possibly during periods of high solar activity.