DEUTERIUM ARRAY MEMO #017

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To: Deuterium Array Group

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Subject: Correlation from beam power

The time critical D1 array receiver host computer calculates the power in a number of beams. For test purposes it is useful to calculate the normalized correlation and phase between antenna ports. In order to avoid separate cross correlation the following algorithm is used:

If the following 4 sums are formed

a+b
a-b
a+ib
a-ib

where *a* and *b* are the complex outputs from ports *a* and *b*. The following beam powers are

$$B_0 = \left|\sum (a+b)\right|^2 = \sum aa^* + \sum bb^* + \sum ab^* + \sum a^* b$$
$$B_1 = \left|\sum (a-b)\right|^2 = \sum aa^* + \sum bb^* - \sum ab^* - \sum a^* b$$
$$B_2 = \left|\sum (a+ib)\right|^2 = \sum aa^* + \sum bb^* - i\sum ab^* + i\sum a^* b$$
$$B_3 = \left|\sum (a-ib)\right|^2 = \sum aa^* + \sum bb^* + i\sum ab^* - i\sum a^* b$$

so that

$$B_0 - B_1 = 2\sum ab^* + 2\sum a^* b = 4\,\mathrm{Re}\sum ab^*$$
$$B_2 - B_3 = -2i\sum ab^* + 2i\sum a^* b = 4\ell m\sum ab^*$$

The beamformer forms a sum with arbitrary amplitude and phase weight for each port so that for each beam

$$b = \sum_i w_i e^{i\phi_i} x_i$$

where w_i is the weight

ϕ_i is the phase

x_i is the complex signal from the i^{th} port

To form the sums needed to get the cross-correlation all the weights are set to zero except those for the 2 ports whose cross-correlation is desired.