DEUTERIUM ARRAY MEMO #023

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To: Deuterium Array Group

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Subject: Useful normalizations

1] Normalized correlation and normalized beam

In interferometry the normalized correlation is useful in the determination of the signal strength from the signal to noise ratio (SNR) which is independent of gain. In this case

$$r = \left\langle xy^* \right\rangle \Big/ \left(\left\langle xx^* \right\rangle \left\langle yy^* \right\rangle \right)^{1/2}$$

where ρ = normalized correlation

If the signals x and y both have the same SNR then

$$SNR = r \big/ (1 - r)$$

while if one has a dominantly high SNR the SNR of the weaker is given by

$$SNR = r^2 \big/ \left(1 - r^2\right)$$

A beam normalization, analogous to normalized correlation, which may be useful in the case of active beamforming is the ratio of the beam power to the sum of the power in each element. In this case

$$b = \sum a_i x_i \sum a_j^* x_j^* \Big/ \sum \left| x_i \right|^2$$

where b = normalized beam

x_i = signal from ith element

a_i = unit magnitude element steering factor

If all the N elements have the same SNR and the source is unresolved and element cross-coupling is ignored:

$$\langle b \rangle = \frac{N\left|x\right|^2 + \left(N^2 - N\right)\left\langle x_i x_j^* \right\rangle}{N\left|x\right|^2}$$

$$= 1 + \left(N-1\right) r = \left(1 + N \times SNR\right)/\left(1 + SNR\right)$$

so that

$$SNR = \left(b-1\right)/\left(N-b\right)$$

2] Effect of ground and sky noise on normalized beam

At first glance one might expect the normalized beam to be greater than 1 but in fact it can be less than one or even zero in an idealized case. For example consider the case of an array with negligible sidelobes with the ground plane pointing at the horizon and beam pointed at the cold sky. In this case

$$b = \left(T_r + T_{sky}\right)/\left(T_r + T_{ground}/2 + T_{sky}/2\right)$$

where T_r = amplifier noise

T_{sky} = sky temperature (assumed uniform)

T_{ground} = ground temperature (assumed uniform)

For T_r = 30 K, T_{sky} =50K, T_{ground} = 300 K b = 0.4

If the beam is pointed at the ground

$$b = \left(T_r + T_{ground}\right)/T_r + T_{ground}/2 + T_{sky}/2$$

which gives b=1.61 in the previous numerical example.

For an array with electronic beam steering the ratio of the normalized beam pointed at the sky to the beam pointed at the ground is $b_{sky}/b_{ground} = \left(T_r + T_{sky}\right)/\left(T_r + T_{ground}\right)$ i.e. the electronic beam steering equivalent of the mechanical "tipping curve." In practice, however, this might only work for an array mechanically aimed at an elevations below the elevation of the electronic beamsteering limit. Also an accounting of the element coupling and integrated array sidelobes is needed. In principle, however, an electronic tipping curve could be used for calibration.

3] Magnitude of coupling effects

The magnitude of the element cross-coupling was measured using dipoles connected directly to a cable with ferrite bead balun. The reference level was determined for the coupling with dipoles right next to each other.

Configuration	Coupling (dB)	Comments
Adjacent dipoles	-24	0.8λ echelon
Diagonal dipoles	-26	$0.8\sqrt{2}l$ side by side
Diagonal dipoles	-30	$0.8\sqrt{2}l$ end to end

The observed correlation between adjacent elements at night or with the sun behind the ground plane is about 0.1. If adjacent dipoles are cross-polarized the correlation drops to less than 0.01. This correlation is due to the combination of the correlation which arises from the incoming radiation, correlated noise from the amplifiers and cross-coupling. The cross-coupling is probably only a minor contributor. A test of the level of correlated noise from the amplifiers in the active antennas can be made by observing the level of an apparent “false” source in the center of the field. By symmetry any cross-coupled noise appears in the central beam.

Consider the effect of correlated noise as follows for the case of a single baseline between elements. In this case

$$x = n_x + a\,y$$
$$y = n_y + a\,x$$

where x and y are the active element outputs, n_x and n_y are the noise signals from each active element. α is the complex coupling factor and includes the complex correlation factor between the emitted noise and the output noise. Noise sources like the gate noise and resistive noise in the input circuitry will tend to be correlated whereas noise in the output circuitry will be uncorrelated.

$$\left\langle xy^* \right\rangle \simeq \left\langle n_x n_y^* \right\rangle + a\left\langle y n_y^* \right\rangle + a^*\left\langle n_x x^* \right\rangle$$

The sum of the second and third terms is real and therefore have the same interferometric phase as a signal arriving perpendicular to the baseline. So far there is no evidence of a false source on axis.