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August 16, 2004

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To: Deuterium Array Group

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Subject: Optimization of D1 detection

Given the revised estimates of the strength of the D1 signals given in memo #52 we will need a substantial amount of observing to detect the line.

1] Equivalent "single dish" integration time

With 24 stations, each with 2 polarizations, we can get the equivalent of 8x24 hours of data each day assuming we observe the anti-center for 4 hours each day. In one year we will get 8 years equivalent integration.

2] Theoretical noise 1-sigma

In one year the 1-sigma noise in each 244 Hz resolution cell is given by

$$(244\times60\times60\times24\times365\times8)^{-1/2} = 4.00 \text{ ppm}$$

where the factor of 8 accounts for the equivalent integration time.

3] Optimum detection method

The optimum detector is a maximum likelihood estimate of the expected line shape of a Gaussian shape with full-width at half power of 20 km/s (21.8 kHz) centered at 0 km/sec with respect to the local standard of rest.

Applying least squares theory

$$y = Ax + \varepsilon$$

where y = column data vector
A = steering matrix
x = signal or parameter column vector
ε= column error or noise vector

$$\hat{x} = \left(A^T A\right)^{-1} A^T y$$

and $C = (x - \hat{x})(x - \hat{x})^T = (A^T A)^{-1} \varepsilon\varepsilon^T$
where C = covariance matrix

If we fit a constant plus a Gaussian the estimated standard deviation in the Gaussian is given by

$$std = \left[N \Big/ \left(N \sum_i g_i^2 - \left(\sum_i g_i \right)^2 \right) \right]^{\frac{1}{2}} \sigma_i$$

where $g_i = e^{-\left[4V_i^2 \log_e 2 / w^2 \right]}$

N = number of spectral points
σ_i = noise in each independent spectral point
V_i = VLSR of i^{th} spectral point
w = full-width half power line width

The noise in the estimate from the square root of the appropriate element of the covariance matrix has a value of 0.15 times the noise in each 244 Hz cell. If we assume that most likely signal strength is 3.7 ppm for an N_D/D_H ratio of 15 ppm then the following table gives the expected signals strength in sigma for observations of one and two years duration.

	Signal detection sigma	
N_D/N_H ppm	1 year	2 years
10	4.1	5.8
15	6.2	8.8
20	8.3	11.7

4] Probability of false detection

In theory a 4 sigma detection has a probability of only 0.03% but in practice a 5 or 6 sigma detection is needed to adequately account for unknown systematic bias. If the N_D/D_H abundance is under 10 ppm we will be unlikely to be able to obtain a reliable detection in the maximum reasonable experiment duration of 2 years.