DEUTERIUM ARRAY MEMO #056

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To: Deuterium Array Group

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Subject: Spectral error estimate in the case of spectral exclusion

In memo #45 I described a method of removing CW RFI by applying a weighted maximum likelihood smoothing in which the spectral channels with RFI receive zero weight. Following the notation of memo 45 the Fourier components of the smoothed spectrum are given by

$$\hat{s} = \left(A^H w A\right)^{-1} A^H w X$$

where X = vector of original spectrum
A = steering or design matrix
s = vector of Fourier series coefficients
w = weight matrix
H = conjugate transpose or Hermitian conjugate
n = error or noise vector

and the expected error in the Fourier coefficients is given by the covariance

$$(\hat{s}-s)(\hat{s}-s)^H = \left(A^H w A\right)^{-1} A^H w n n^H w^H A \left(A^H w A\right)^{-1} = \left(A^H w A\right)^{-1}$$

since $\left(nn^H\right)_{ij} = 1$ for $i=j$

since $= 0$ for $i \neq j$

and $ww^H = w$ since $w_{ii} = 1 \text{ or } 0$

the spectrum of the estimated error is given by the diagonal elements of the transformed covariance matrix

$$\sigma_i^2 = \left(A(\hat{s}-s)(\hat{s}-s)^H A^H\right)_{ii} = \left(A\left(A^H w A\right)^{-1} A^H\right)_{ii} \sigma_0^2$$

where $\sigma_0 = (bT)^{-1/2}$

b = original spectral resolution = 244 Hz
T = integration time

Where there are many adjacent points in the original spectrum for which there is RFI so that their weight is zero there will be an increase in the spectrum of the error in the smoothed spectrum. In fact if there are regions of the downweighted spectrum which span the resolution of the smoothing the estimated error will become large or infinite in these regions. In practice, when computing the expected error, it is advisable to first test the matrix inversion (or equivalently the rank of the matrix) to make sure the matrix is not singular.

When spectral RFI exclusion is applied each day and then the smoothed spectra are averaged, following Doppler steering, for many days the error spectrum from the transformed covariance matrix can also be averaged after appropriate weighting to provide information on the uniformity of the errors in the average spectrum as a functions of VLSR. The error spectra for the individual days can also be used to filter out any case where there are large peaks in the error spectrum.