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To: Deuterium Array Group

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Subject: The statistics of cumulative spectra

For the D1 experiment we are continuously accumulating spectra and examining the SNR in order to:

 1] See if we have a significant detection
 2] Estimate how much longer we need to integrate in order to obtain a significant detection.

If the noise has purely Gaussian statistics without systematics the probability of false detection for a SNR is given by

$$PE = \frac{1}{\sqrt{2\pi}} \int_{s}^{\infty} e^{-x^2/2} dx = \frac{1}{2} erfc\left(s/\sqrt{2}\right)$$

$$\approx \frac{1}{s} \frac{e^{-s^2/2}}{\sqrt{2\pi}} \quad \text{when } s \gg 1$$

PE is approximately 0.15% for $s = 3$.

Owing to the presence of systematics more confidence can be gained in a detection by integrating longer to improve the SNR. However at low SNR the added integration may not increase the calculated SNR owing to the noise. At an SNR of 2.0 there is about a 15% chance that the SNR will not increase by doubling the integration even though a real signal is present. Figure 1 shows the probability of false detection and the probability that doubling the integration will fail to improve the SNR as a function of the true SNR. We define the true SNR is signal/sigma where the signal value is the actual signal present and what would be measured with the infinite integration while sigma is the calculated noise.

The probability that the SNR will fail to increase in doubling the integration PD is given by

$$PD = \int_{-\infty}^{+\infty} \frac{1}{2} erfc\left(\left(\sqrt{2}-1\right)(s+y)\right) \frac{e^{-y^2/2}}{\sqrt{2\pi}} dy$$

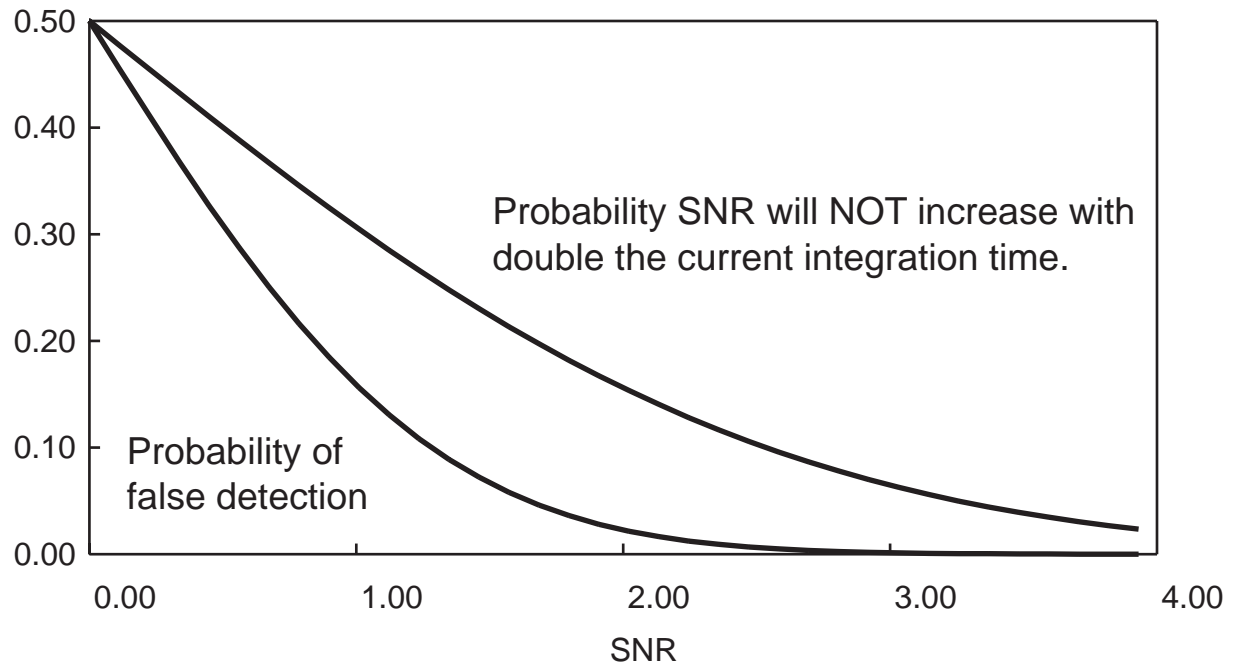Figure 1 is a plot of PE and PD. Figure 2 shows the progression of SNR for 3 trials of simulated data.

Figure 1. Low SNR statistics

Figure 2.