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To: Deuterium Array Group

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Subject: Alternative method of CW RFI amelioration

If the final objective is a least squares fitting of the expected D1 profile to the average spectrum there is an alternate method for the amelioration of CW RFI. In this method the expected profile is separately fit to each stations average spectrum for each day and then the weighted average of the amplitudes of the profile taken as an equivalent measure of the profile amplitude for the entire dataset.

Following the notation of memo #54

$$\hat{x}_k = \left(A^T A\right)^{-1} A^T y_k$$

where k is an index for a subset of the data. From this it follows that

$$\hat{x}_{AV} = \left(A^T A\right)^{-1} \sum_0^{K-1} y_k \Big/ K = \frac{1}{K} \sum_0^{K-1} \hat{x}_k$$

CW RFI can be ameliorated by using a weighted least squares

$$\hat{x} = \left(A^T w A\right)^{-1} A^T w_y$$

where the weight is zero for frequency channels with RFI.

In averaging the fitted parameters the optimum average is a weighted average. The standard deviation in the n^th parameters

$$\sigma = \left[\left(A^T w A\right)_{nn}^{-1}\right]^{1/2} \sigma_0$$

where $\sigma_0 = \left(BT\right)^{-1/2}$
B = resolution = 244.14 Hz

T = integration sec

The optimum weighted average

$$x_{av} = \sum w_k x_k \Big/ \sum w_k$$

where $w_k = 1 \big/ \sigma_k^2$

$$\sigma_{av}^2 = \sum_k w^2 \sigma_k^2 \Big/ \sum_k w^2 = 1 \Big/ \sum_k 1 \big/ \sigma_k^2$$

There are two advantages of fitting an expected profile to each station day. First the need to fit a intermediate Fourier series is not required and second there is more flexibility in averaging subsets of the data. The fitted parameters include, a constant, the expected D1 profile and several added terms of a polynomial. For example

$$s(w) = a_0 + a_1 w + a_2 w^2 + a_3 w^3 + a_4 w^4 + a_5 d(w)$$
$$x^T = [a_0, a_1, a_2, a_3, a_4, a_5]$$

$d(w)$ = expected line profile vs frequency or velocity

The larger the number of polynomial coefficients the more systematic bandpass curvature, etc. that can be removed from the fit but the standard deviation in the fit will increase as the parameters become more highly correlated. In fact the matrix inversion may be marginal for more than 6 coefficients if many frequencies are downweighted. The best choice may be around 5 polynomial terms plus the expected D1 profile.

Tests of splitting data into 2 sets from day 2004-190 to 2004-326

Data	D1 amplitude ppm	SNR	Integration yr
Days 190-259	5.4	3.2	1.52
Days 260-326	3.0	1.8	1.57
“a” pol	6.2	3.7	1.55
“b” pol	2.1	1.3	1.54

Tests of the use of reference spectra:

Signal	Ref	D1 amplitude ppm	SNR	Integration yr
G183	None	3.0	2.5	3.1
G195	None	-0.7	0.4	1.4
G171	None	3.5	3.1	3.4
R06183	None	-1.5	1.2	2.8
R12183	None	-0.7	0.6	2.8

R18183	None	0.7	0.6	3.3
G	None	2.5	3.4	7.9
R	None	-0.4	0.6	9.0
G183	Yes	4.2	3.5	3.1
G195	Yes	1.1	0.7	1.4
G171	Yes	2.0	1.8	3.4
R06183	Yes	0.3	0.3	2.8
R12183	Yes	0.2	0.1	2.8
R18183	Yes	1.5	1.3	3.3
G	Yes	2.7	3.7	7.9
R	Yes	0.7	1.0	9.0

The reference spectrum used was that of the average of all the dipoles over the same time span and with the same RFI excision as the beam data. This provides a sky reference with a relatively broad beam centered at the zenith. It has the advantage that the noise is only increased by $\left(1+\frac{1}{24}\right)^{\frac{1}{2}}=1.02$ or 2%, where as using another single beam as reference increases the noise by $2^{\frac{1}{2}}=1.414$ or 40%. Using a reference which is an average of beams or include data from the reference which is outside the time span of the signal data reduces the added noise but may be as effective in reducing systematics.

These results are all statistically consistent with a marginal detection of D1 signal of about 3 ppm and systematic influences less than about 1 ppm.