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Subject: Methods of smoothing spectra

The most common method of changing the resolution of a FFT spectrometer is to change the FFT size and/or apply a windowing function to the time samples. Changing the FFT size for the fixed length blocks of samples is fine but applying a windowing function in the time domain is not optimum as it also reduces the effective integration so that the noise is increased compared with the optimum system for which the one sigma fractional noise is

$$(b\tau)^{-\frac{1}{2}}$$

where b = resolution bandwidth
τ = integration time

The signal to noise ratio (SNR) could be improved by adding windowed data blocks which are time shifted by half a block. However this is still suboptimal and doubles the number of required computing operations. A better method is to increase the block length and to apply a smoothing convolution to the spectrum to obtain the derived resolutions. This is more efficient as the required operations per unit time for a FFT increase with the logarithm of the block length. The unweighted FFT spectrum is equivalent to the autocorrelation spectrum derived by weighting the autocorrelation with the triangular window function. In this case the noise bandwidth is equal to one FFT bin.

The resolution bandwidth can be increased by convolving the spectrum with a "smoothing" function which equivalent to multiplying the autocorrelation by the transform of the smoothing function. For example, using the following smoothing

$$sm(w) = e^{-w^2\pi^2/(2a^2)}$$

increased the noise bandwidth by the factor, *a*, so that the one sigma noise becomes

$$(ab\tau)^{-\frac{1}{2}}$$

where b is the spacing between FFT bins. The rejection of out signals outside the resolution window increased by a^2 or 6 dB by each factor of 2. If the resolution in Hz is kept constant. For example, going from a 2048 FFT to a 16348 increases the computing by 27% for a 18 dB improvement in spectral leakage.