MASSACHUSETTS INSTITUTE OF TECHNOLOGY
HAYSTACK OBSERVATORY
WESTFORD, MASSACHUSETTS 01886

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Telephone: 617-715-5533

To: EDGES Group

From: Alan E.E. Rogers

Subject: Formula for fractional ripple from scattering

A concern that scattering from objects near the antenna might be producing correlated signals from strong compact radio sources in a manner similar to the "Sea Interferometer" was raised in memo 335 when looking at the fine frequency structure and rapid changes in the residuals vs GHA. In the Sea interferometer a single antenna is used to form an interferometer which used these correlations to map radio sources with a single antenna in the 1950s as discussed in memo 340. Concern that scattering might be responsible for the large residuals near transit of the Galactic center were raised and studied in memos 340, 341, 342, 343, 344 and 345 and the effects of nearby objects on the antenna beam were checked in memo 348. Effects of an uneven ground plane were studied in memos 355, 356 and 357. As an added tool to compare the FEKO results an analytic approximation based on isotropic scattering is derived below:

The spectrum of the radio sky in the 50 – 200 MHz frequency band is relatively smooth because the only spectral lines are recombination lines which are very weak. When the sky is observed with a small antenna with low beam chromaticity on an infinite plane ground plane the observed spectrum is the average of many continuum radio sources and will be smooth if the receiver and antenna are well calibrated. However, sky noise can be scattered from imperfections in the ground plane and surrounding objects. In this case the signals from the radio sources in the sky which are scattered can also enter the antenna.

If the scatter from a single point source is considered the power, Psource, is given by

$$Psource = AsourceF \qquad (1)$$

where Asource is the effective aperture (area) of the antenna in the direction of the radio source with flux, F, while the power, Pobject, scattered from an object with radar cross section, X, is

$$Pobject = XF \qquad (2)$$

if the power is scattered isotropically the power, Pscat, received by the antenna at a distance, d, from the antenna is

$$Pscat = AscatF\, X/(4\pi d^2) \qquad (3)$$

where Ascat is the effective aperture (area) of the antenna in the direction of the scattering object on the assumption that the direct and scattered signals are perfectly correlated the combined complex voltage, v is given by

$$v = Psource^{1/2} + e^{i\omega\tau}Pscat^{1/2} \quad (4)$$

and the power is vv^*which for small Pscat relative to Psource is equal to

$$Power = Psource(1 + 2\cos(\omega\tau)(Ascat/Asource)^{1/2}(X/(4\pi d^2))^{1/2}) \quad (5)$$

where $\omega = 2\pi frequency$

and τ= the delay of the scattered signal relative to the direct signal which equals

$$= d\,[1 - \cos(el)\cos(azd)]/c \quad (6)$$

where d is the distance of the scattering object from the antenna and azd = the difference in the azimuth of the radio source and the scattering object as seen for the antenna and c is the velocity of light.

The ripple fraction is given by

$$ripple = \pi^{-1/2}\cos(\omega\tau)(Gscat/Gsource)^{1/2}\,X^{1/2}/d \quad (7)$$

where the ratio of gains has been substituted for the ratio of effective aperture (area).

As an example the flux of Cas A at 100 MHz the flux is 1.4e4 J which results in an antenna temperature of 23 K for Gsource = 8 dB. Then the substitution of Gscat = -23 dB for the antenna gain in the direction of a scattering object with radar cross section of X =10m^2 at d = 75m results in a peak to peak ripple of 31 mK using equation 7. It is noted that the ripple decreases in proportion to the inverse of the distance but at large distances the source may be resolved by the projected interferometric baseline and the ripple period shortened so that it is smoothed by the spectral resolution. It should also be noted the scattering by multiple objects will generate a complex waveform. The ripple period which is given in equation 6 changes with LST and can become very long for a radio source which in low in elevation at the azimuth of the scattering object.

The power fraction returned to a VNA from the scattering object is given by

$$Pobject = GscatX\,Asource/((4\pi)^2 d^4) = Gscat^2 X\,\lambda^2/((4\pi)^3 d^4) \quad (8)$$

and the resulting voltage or equivalently the change in antenna s11, δs11, due to the scatter is

$$\delta s11 = GscatX^{1/2}\lambda\,e^{i\omega 2d/c}/((4\pi)^{3/2}d^2) \quad (9)$$

and the resulting ripple in the sky noise from derivative of $1 - |s11|^2$

$$ripple = 2Tsky|s11|\cos(\omega 2\,d/c)GscatX^{1/2}\,\lambda/((4\pi)^{3/2}d^2) \quad (10)$$

The magnitude of δs11 is 2e-7 and for Tsky of 3000 K and antenna s11 of -10 dB the ripple in the sky noise spectrum due to the ripples in s11 is only 0.6 mK peak to peak and the ripple in the -10 dB s11 is only 1e-5 dB peak to peak.

In summary equation 7 predicts a much larger effect on the observed sky spectrum than the effects of scatter have on the antenna s11 predicted by equation 10 which also effect the observed sky spectrum.

Reflections from the electronics hut used for the receiver backend and other electronics has long been a concern as ripples have been observed in the lowband 1 data for which the hut 50 meters is west of the antenna. Figure 1 shows a simulation of the residuals made using FEKO to produce the beam with the inclusion of the hut on an infinite PEC ground plane for simplicity and rapid computation. 5-physical terms are used to remove the foreground based on the convolution of the beam with the Haslam map.

Figure 2 shows the residuals are obtained from the spectra which have now been integrated over 1 hour blocks instead of the instantaneous spectrum at each hour. The averaging greatly reduces the effects of the scattering because of the rapid change with GHA that smear out the effects. Figure 3 shows that results of the scatter which produce fine angular structure in the beam have a much smaller effect when convolved with the Guzman sky map which has lower angular resolution angular resolution than the Haslam map. This demonstrates that scattering effects observed in real data cannot be removed with beam correction made with a sky map that has insufficient angular resolution to accurately represent the true sky. This emphasizes the importance that global 21-cm systems operate in an environment free of objects which can produce scattering effects and have a large uniform ground plane or operate in space away from other objects.

Figure 4 shows the modeling scattering effects made by adding the ripple fraction approximation to the the beam convolution. In this case the beam is an analytic beam or a beam modeled with FEKO without the scattering object. This may be useful in assessing the effects of potential scattering objects without the need for a complex FEKO model which may not be practical owing to the very long compute time or limitations of the accuracy in a large complex model. In order to obtain a result comparable in magnitude to the scattering residuals in Figure 1 the scattering cross section X in equation 7 has been increased from 5 to 35 m^2 to account for the scattering using FEKO being more directed at low elevations than isotropic as shown beam pattern difference in Figure 6.

Figure 5 shows a comparison of a simulation of the scatter from the hut with the lowband 1 observations in the GHA range where the effects of the scattering from the hut is the strongest based on the results in Figure 1.

A test using an artificial sky map with a point source at the Galactic center shows that the main source which produces the effects of the hut on the residuals is the Galactic center.

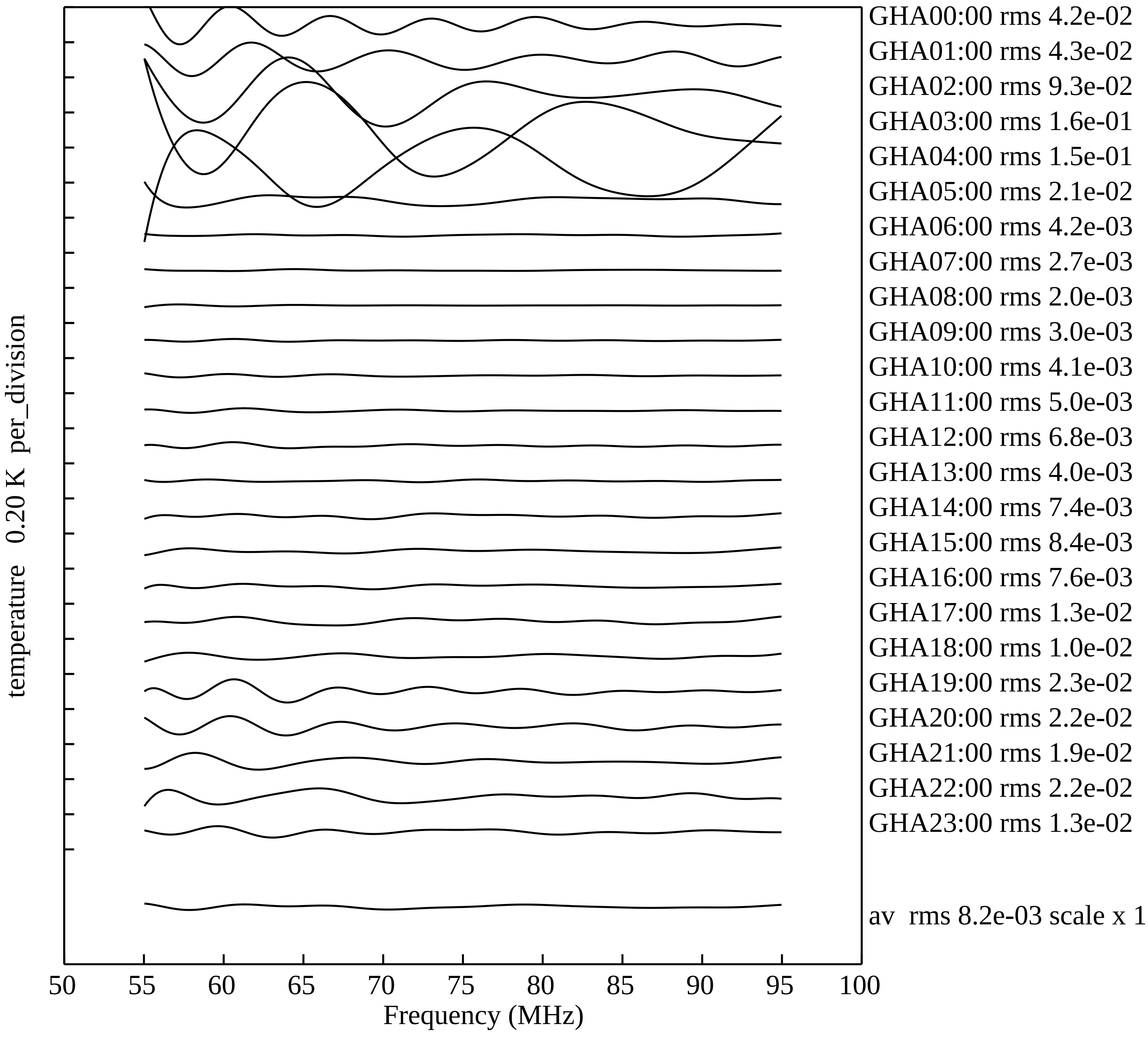

Figure 1. Simulations of residuals with 5-physical terms removed using lowband beam derived using FEKO with infinite PEC ground plane with the hut 50 meters west of the antenna which is pointed north. Beam convolution used the Haslam map. Beam correction used the FEKO beam on PEC ground without the hut.

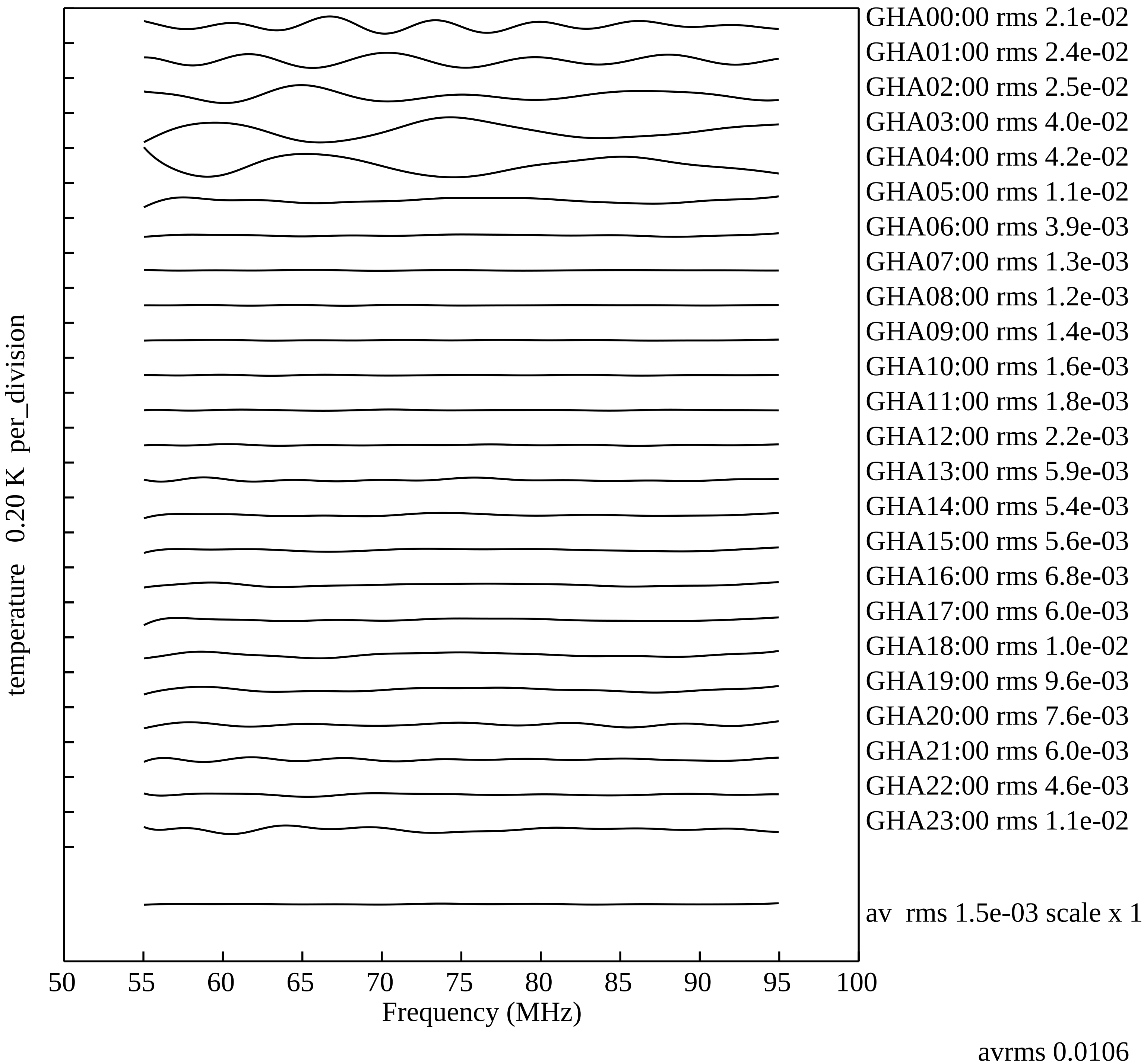

Figure 2. Same as Figure 1 but with 1 hour integration of the spectra at each GHA.

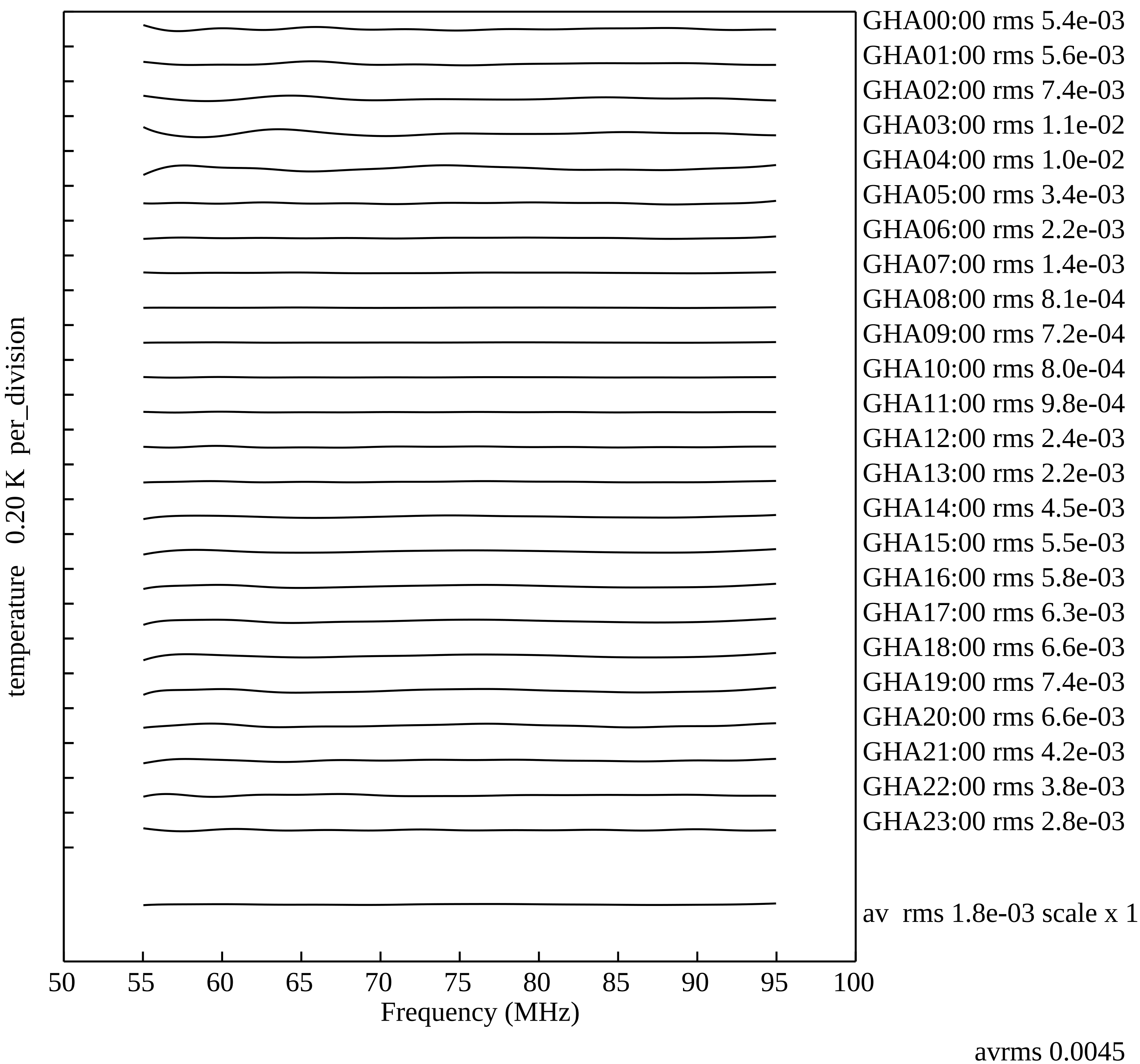

Figure 3. Same as Figure 1 using Guzman map.

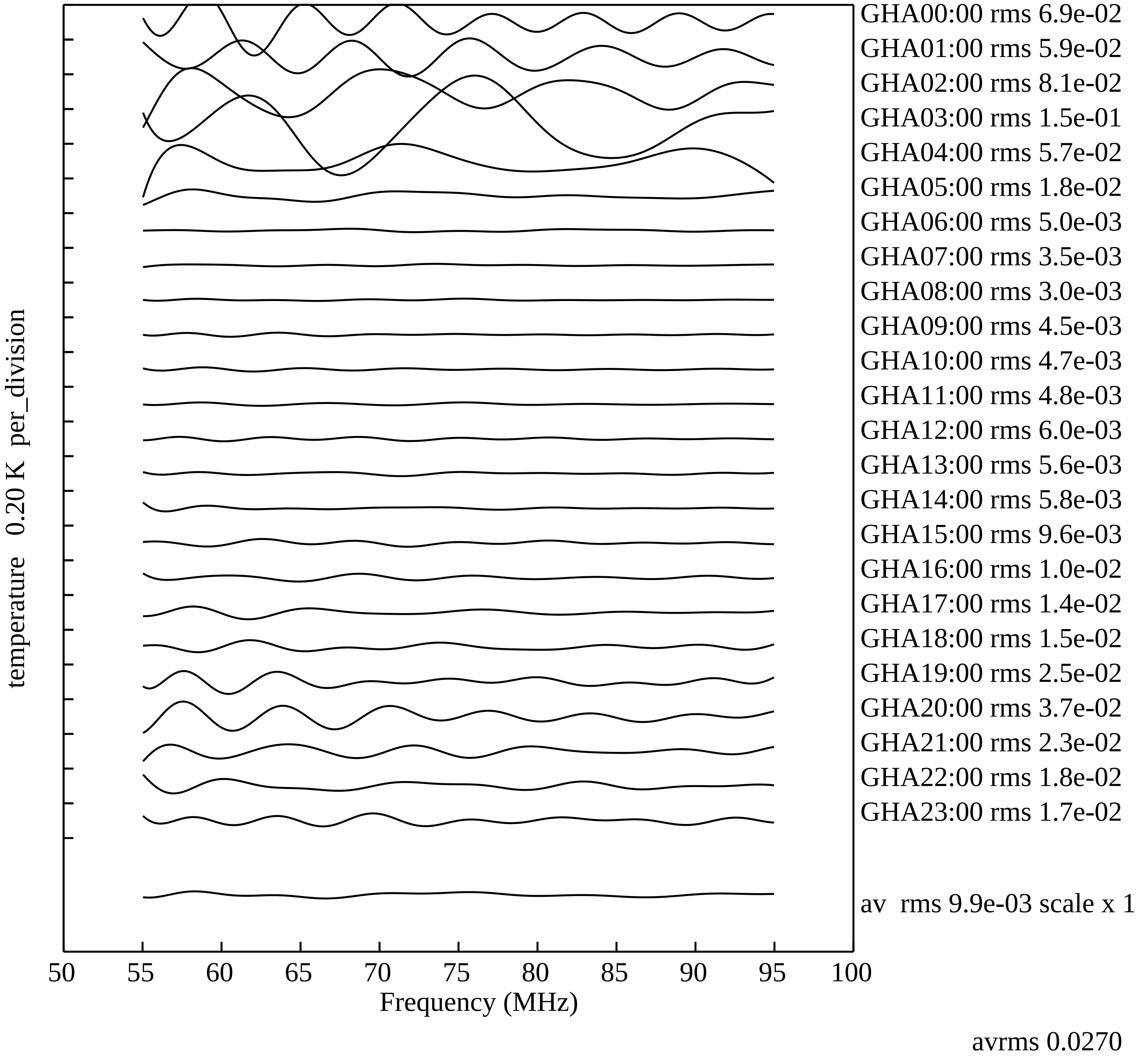

Figure 4. Simulations of residuals with 5-physical terms removed using lowband beam derived using FEKO with infinite PEC ground plane. The effect of the hut was obtained by including the fractional ripple from equation 7 into the beam convolution using the Haslam map.

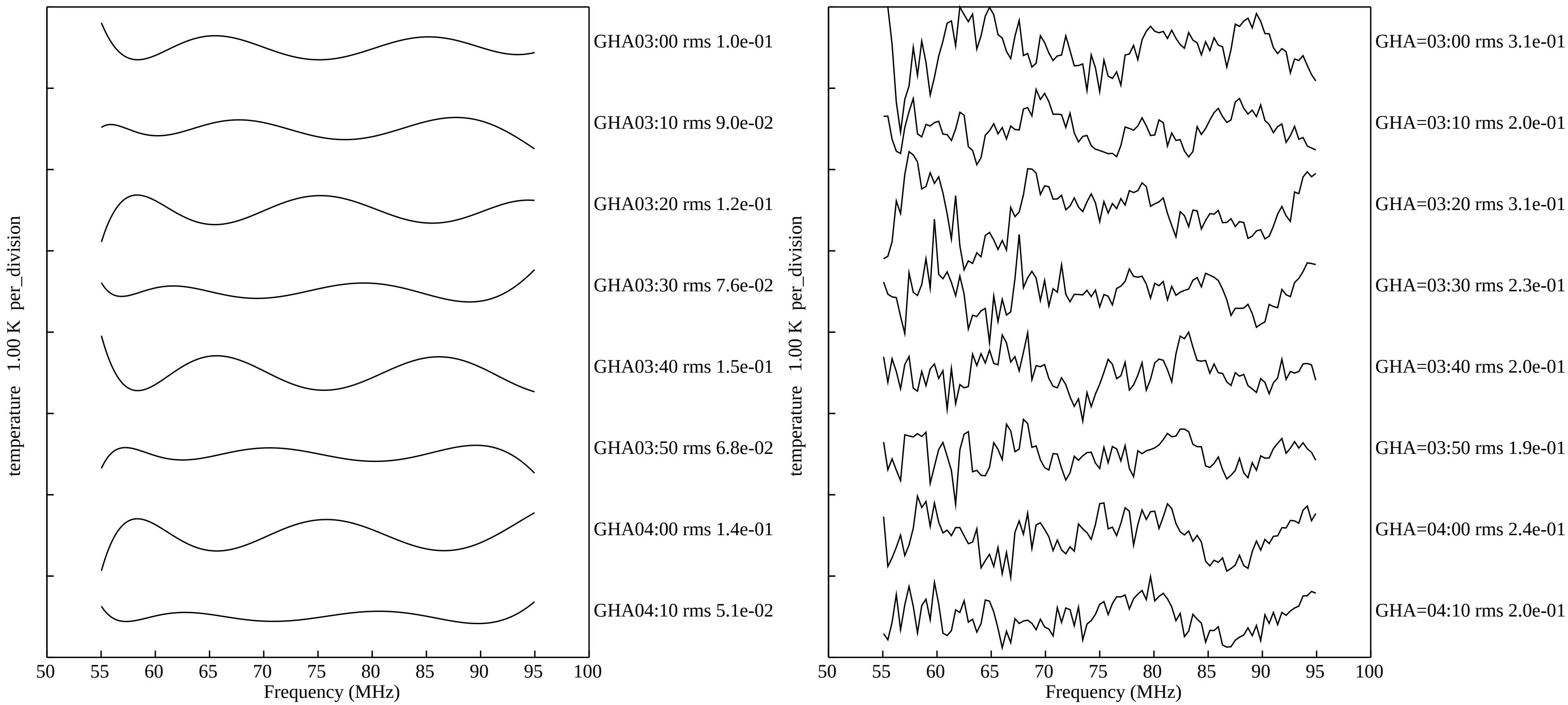

Figure 5. A comparison of the simulation of the effect of the hut as in Figure 1 using simulated spectra on the left for every 10 minutes for the GHA range, where the effects of the hut are the largest, compared with the spectra from lowband 1 taken from 2016 day 250 to 2017 day 95 on the right.

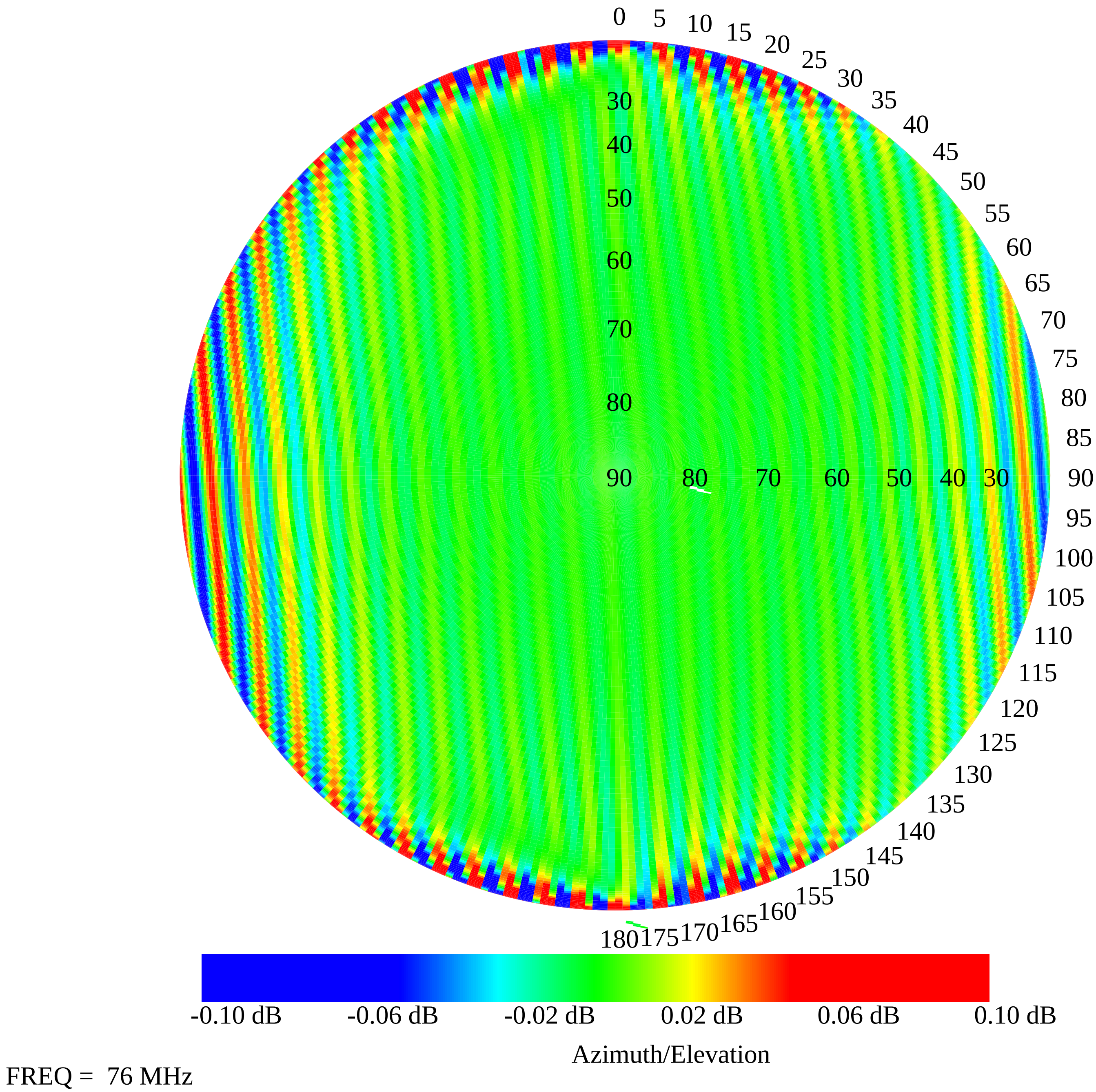

Figure 6. Difference of the antenna beam patterns at 76 MHz with and without the hut from FEKO models on an infinite PEC ground plane.