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Holographers
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Subject: Diffraction of subreflector and its support struts
At Mehdi Zarghamee's suggestion, I have now included the subreflector and its support struts in the diffraction model.

## 1] Holographic phases and Scott/Ryle Correction

As shown in a previous memo (dated 26 November 1990), the path length d increase from an element at $(\mathrm{x}, \mathrm{y})$ on the antenna surface is

$$
\begin{equation*}
d=x \theta+\left(a+\left(x^{2}+y^{2}\right) /(4 f)\right) \theta^{2} / 2 \tag{1}
\end{equation*}
$$

$$
\begin{array}{rlr}
\text { where a } & =\text { distance from the vertex to rotation axis } \\
f & =\text { focal length } \\
\Theta & =\text { angle scanned in the } \mathrm{x} \text { direction }
\end{array}
$$

For an element on the radome which projects onto ( $x, y$ ) in the aperture plane

$$
\begin{equation*}
d=x \theta-\left(a+\left(x^{2}+y^{2}\right) /(4 f)\right) \theta^{2} / 2 \tag{2}
\end{equation*}
$$

for a ray which passes through the element. Further for an element on the subreflector or support strut which projects onto ( $x, y$ )

$$
\begin{equation*}
d=x \theta+b \theta^{2} \tag{3}
\end{equation*}
$$

where $b=$ the distance from the element to a plane through the antenna rotation axis intersection and parallel to the aperture plane.

For a diffracting element on the strut in the path between the prime focus and the main reflector, the path length is the same as for an element on the surface given in Equation 1. The geometry which gives rise to the path length is shown in Figure 1.

The path length (higher order terms being neglected) is used to derive the complex "beam" $\mathbb{B}(\mathrm{x}, \mathrm{y})$ function which to convolve the diffraction in order to account for the holographic scanning.

$$
\begin{gather*}
B(x)=\int_{-\Phi}^{+\Phi} e^{i k d} d \theta  \tag{4}\\
\text { and } B(x, y)=B(x) B(y) \tag{5}
\end{gather*}
$$

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where \Phi = half-angle scanned
and k = 2\pi/\lambda
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## 2] Diffraction scaling

The diffraction from the radome and from struts on the inner portion of the aperture plane involve only plane incident wavefronts and unit distance in the aperture plane scales to the same distance in the diffraction plane. However, diffraction of the elements in the path between the prime focus and the main reflector involve scaling or magnification changes as illustrated in Figure 2. The magnification is given by

$$
\begin{equation*}
(g+h) / g \tag{6}
\end{equation*}
$$

where $g$ is the distance from the prime focus to the diffracting element and $h$ is the distance from the diffracting element to the main reflector surface.

3] Fresnel wavefront delay
For the diffraction of elements between the distance transmission and the main reflector, the added distance for paths which deviate from the central ray is

$$
\begin{equation*}
s^{2} /(2 h) \tag{7}
\end{equation*}
$$

where $h=$ distance from the diffracting element and the main reflector surface
$s \quad=\quad$ perpendicular distance from diffracting element to the central ray
For diffraction of the spherical wavefront from the prime focus the diffraction path is

$$
\begin{equation*}
s^{2}(g+h) /(2 g h) \tag{8}
\end{equation*}
$$

Figure 3 shows the amplitude of the diffraction pattern. Note that the thin support rods are only barely evident when they align with the polarization of the satellite signal used for the holography.

At this time the results, while in qualitative agreement with the holography, have not yet been quantitatively compared with the holography. Also, Bob Cady and I are still carefully checking the geometry (size and location of struts, rods, etc.) assumed in the software with the antenna drawings.

$\left.\square_{d=x \Theta+\left(a+\left(x^{2}+y\right.\right.}{ }^{3} /(4 f)\right) \theta \xi^{2}$
CASE 1. ELEMENT ON SURFACE

$\square_{d=x \theta-\left(a+\left(x^{2}+y\right) /(4 f)\right) \theta^{2} / 2}$
CASE 2. ELEMENT ON RADOME
Figure 1. PATH LENGTH

$d=x \Theta+b \Theta^{2}$
CASE 3. ELEMENT ON STRUT

$\square d=x \Theta+\left(a+\left(x^{2}+y^{3}\right) /(4 f)\right) \theta z_{2}$
CASE 4. ELEMENT ON STRUT
CHANGE WITH SCAN ANGLE



