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31 March 1992

To: Holographers

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Subject: Holography resolution functions — Addenda to 25 March 1992 memo

Addendum 1. A serious oversight in my earlier memo was the failure to point out and to quantify a second effect of convolving the "true" map with a resolution function: not only are narrow features broadened (see Figure 1 in the memo), but their peak amplitudes are reduced.

To compute the effect on the amplitudes, the resolution function used in eq. (2) should be that obtained directly from eq. (3), without normalization to unity at the origin. These unnormalized equivalents to eqs. (4) and (5) are

$$R_s(x,y) = 4u_{max}^2 \frac{\sin kxu_{max}}{kxu_{max}} \frac{\sin kyu_{max}}{kyu_{max}} \tag{4'}$$

$$R_c(x,y) = \pi u_{max}^2 \, 2 \frac{J_1(kru_{max})}{kru_{max}} \tag{5'}$$

The new factors in front of the sinc and Bessel functions are simply the areas of the windows in the (u,v) plane.

As an example, Figure 2 shows the effect of convolving circular gaussian functions of various widths with the circular resolution function for our 91×91 maps. The peak amplitude of a gaussian (FWHM = Δr) after convolution with $R_c(x,y)$, relative to its original amplitude, is $1 - \exp(-k^2 u_{max}^2 \Delta r^2 / 16 \ln 2)$. For our 91×91 maps, this ratio is $1 - \exp(-\Delta r^2/(53.6 \text{ cm})^2)$.

Addendum 2. The FWHM of the circular resolution function in Figure 1 is 71 cm, and yet we have been claiming that 91×91 maps have 50-cm resolution. So what *is* our map resolution?

There is no single answer to this question, because there is no single, universally accepted definition of "resolution". The Rayleigh criterion used in optics specifies the resolution as the distance between the principal maximum and first minimum of the intensity distribution, which is 62 cm for the 91×91 maps (see Fig. 1). Alternatives include the FWHM of the resolution function (71 cm) and the cutoff wavelength of the frequency response function (101 cm). The 50 cm number came about by the following argument: If we sample the antenna beam at the Nyquist (angular) rate at N points, then we will have N independent values of amplitude and phase in the map plane. For $N = 91$ and a map size of $\lambda/\Delta u = 45.4$ m, the average (in some ill-defined sense) distance between independent map points is $(45.4 \text{ m})/91 = 50$ cm.

The point is that the *definition* of resolution is arbitrary, to a large extent, but the effect of finite resolution on the maps is perfectly well-defined (see Figs. 1 and 2, for instance).

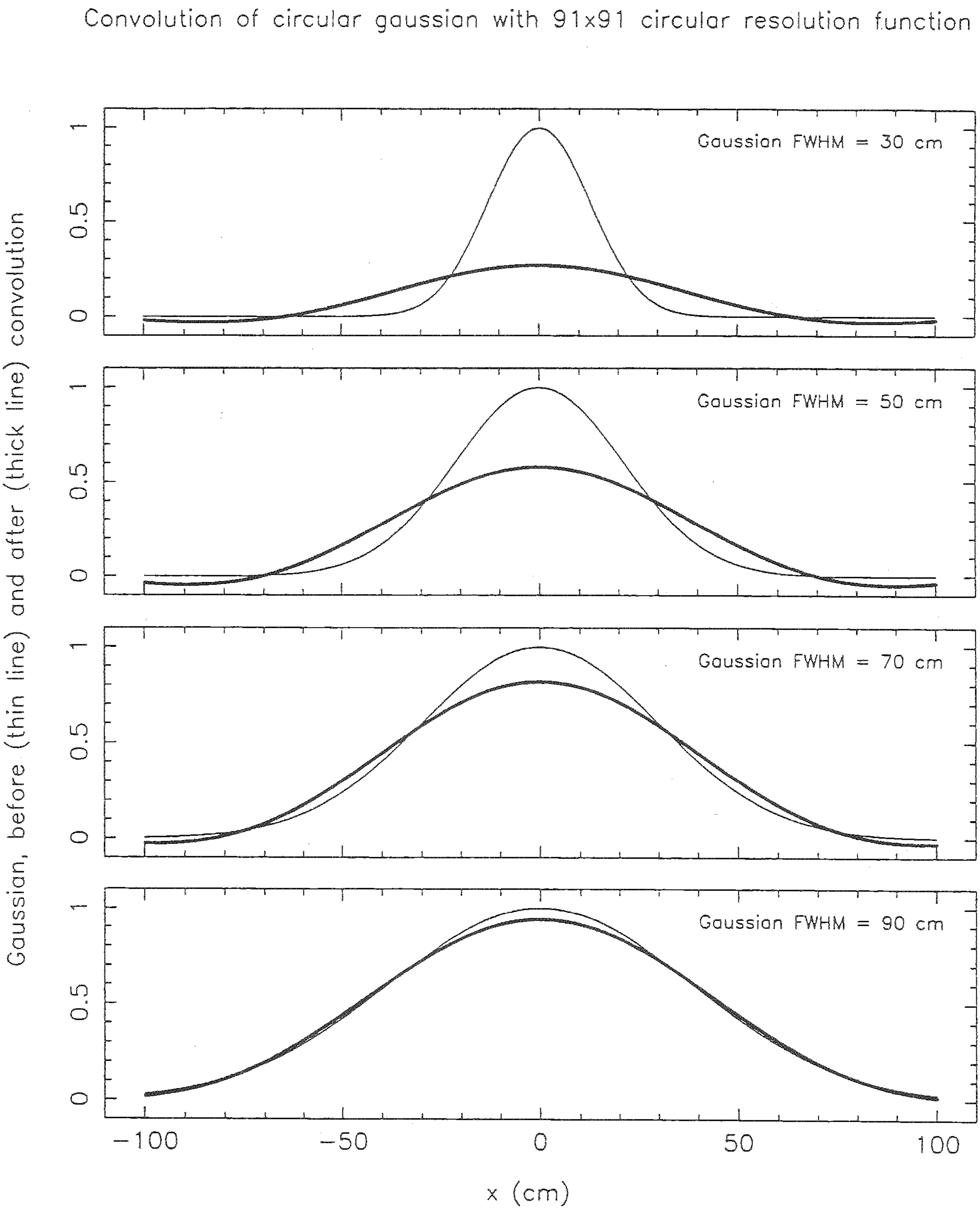

Figure 2. Cross-section through circular gaussian function, before (thin line) and after (thick line) convolution with 91×91 circular resolution function [eq. (5′), with $k = 2\pi/(2.54\ \text{cm})$ and $u_{max} = 0.0251$].