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To: Millimeter-wave VLBI Group

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Subject; Numerical solution of Maxwell's equations in parallel plate polarizer

## Introduction

Shep and I have searched the literature for solutions of the parallel plate polarizer. An analytic solution was performed by Nobel Laureate Julian Schwinger when he worked at the radiation laboratory during WWII. Unfortunately, this solution is very complex and difficult to interpret. Also, only the results of the solution are given in the Waveguide handbook by Marcuvitz. In order to check Schwinger's results, those of Carlson and Heins (1947), and Lengyel (1950), we have used a numerical method made practical by the availability of modern computer power.

## Numerical solution method

If we use the coordinate system shown in Figure 1 the electric field for the $m^{th}$ mode is given by

$$Ey = \cos(x \pi m)\, e^{\pm K_m z}\, e^{-iwt} \tag{1}$$

where

$$K_m = \left[ \left( \frac{\pi m}{d} \right)^2 - \left( \frac{2\pi}{\lambda} \right)^2 \right]^{1/2} \tag{2}$$

$d$ = *separation between the plates*

$m = 0$ is a plane $TE$ wave, $m = 1$ is the only propagating mode between the plates. For modes $M \geq 2$, $K_m$ are real, and the modes are evanescent.

Since

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \tag{3}$$

the components of magnetic field are

$$Hx = -\left[\frac{1}{iw\mu_0}\right]\frac{\partial Ey}{\partial z} \tag{4}$$

$$Hz = \left[\frac{1}{iw\mu_0}\right]\frac{\partial Ey}{\partial x} \tag{5}$$

For entering the parallel plates the electric field inside and outside the plates must be continuous except at metal boundaries so that

$$T\cos \pi x + \sum_{odd} d_m \cos m\pi x = (1 + R) + \sum_{even} c_m \cos mx\pi \tag{6}$$

| | | |
|---|---|---|
| where T | = | amplitude of mode propagated into polarizer |
| R | = | amplitude of reflected plane wave |
| $d_m$ for *m odd* | = | amplitude of evanescent modes between plates |
| $c_m$ for *m even* | = | amplitude of evanescent modes outside plates |

Only odd modes (as defined by equation 2) can exist between plates because the electric field must be zero at $x = \pm ½$. Only even modes can exist outside plates because the electric field must repeat in a continuous manner with a periodicity of 1 - as illustrated in figure 2.

The corresponding equation for magnetic field component $(Hx)$ is

$$(K_1/K_o)T \cos\pi x - \sum_{odd} \frac{d_m K_m \cos m\pi x}{K_o} = (1-R) + \sum_{even} \frac{c_m K_m \cos m\pi x}{K_o} \tag{7}$$

If equation (6) is satisfied then the equation for $Hz$ will also be satisfied - so the problem is to simultaneously satisfy equations (6) and (7). This is easily accomplished using matrix methods to find $c_m$ and $d_m$. Another set of equations is needed for the boundary conditions of radiation leaving the polarizer.

These are:

$$(1 + R) \cos \pi x + \sum_{odd} a_m \cos m\pi x = T + \sum_{even} b_m \cos mx\pi \tag{8}$$

and

$$(K_1/K_o)\ (1-R)\cos\pi x + \sum_{odd} \frac{a_m K_m \cos m\pi x}{K_o} = T - \sum_{even} \frac{b_m K_m \cos m\pi x}{K_o} \tag{9}$$

These equations can be solved for 2N modes using a range of N discrete values of x from zero to ((N-1)/2N). For the complete path through the polarizer the transmitted waves are cascaded and the reflections added with appropriate propagation phases. The numerical solutions are performed using the MATLAB program given in appendix 1. The loss and differential phase is plotted in Figure 3 for the dimensions of the polarizer built for use on Haystack at 86 GHz. The solution includes reflections to the second order. The differential phase shift differs from the simple theory which neglects reflections and the effects of energy storage in the evanescent modes. These results are in good agreement with those of Lengyel. The Schwinger solution from Marcuvitz (as interpreted by AEER) gives a reflection voltage that agrees well with our results but gives a phase shift that disagrees with our result and the result of Lengyel. We presume that our application of Schwinger's solution to calculate the phase path is incorrect.

Ohmic losses

If the evanescent modes are neglected the polarizer loss due to the finite conductivity of the metal vanes is given by

$$\frac{8x_0 a}{d}\ \left(\frac{\lambda}{\lambda c}\right)^2 L \quad nepers \tag{10}$$

where $$x_0 = 1/2 \left[\frac{\pi \varepsilon}{\sigma}\right]^{1/2} \tag{11}$$

$$a = \nu^{1/2} / \left(1-(\lambda/\lambda c)^2\right)^{1/2} \tag{12}$$

$$\lambda c = 2d \tag{13}$$

and $L$ = *length of polarizer*

The formula above is that of a waveguide of unlimited height. Evaluating the above with

$L$ = 5.2 mm
$d$ = 3.2 mm
$\lambda$ = 3.5 mm

we obtain a loss of 0.2% for brass (4 times the resistance of copper). When evanescent modes are included the losses are increased by about 50% by the enhanced currents near the edges of the plates. In practice, waveguide loss in the millimeter range often runs twice the theoretical owing to surface irregularities and defects on the scale of the skin depth ($\approx$1 micron). Thus a conservative estimate for ohmic loss is about 1%.

## Plate thickness

The numerical solution can easily be made for the case of finite plate thickness by using the appropriate wave $K$ values (modified by the decreased separation inside the polarizer) on each side of the boundary and imposing the constraint of zero tangential E-field on the front edge of the plates. Figure 4 shows the solutions for the E- and H-field which satisfy the boundary conditions. The currents in the plates which produce the H-field discontinuity concentrate at the corners of the plate edges. As the plate thickness is increased the reflection coefficients are increased and changed producing added loss. Table 1 gives the results for various thicknesses.

| Thickness mm | Thickness Percent | Loss without scatter | Loss with 100% scatter |
|---|---|---|---|
| 0 | 0% | 0.01% | 1.6% |
| 0.32 | 10% | 0.1% | 2.5% |
| 0.64 | 20% | 3.4% | 4% |
| 1.28 | 40% | 33% | 20% |

Table 1. Frequency = 86 GHz; $d = 3.2\ mm$; $1 = 5.2\ mm$
Reflection losses for various plate thicknesses

The loss is given for 2 cases. In the first case it is assumed that there are only the modes appropriate for an infinite number of plates while in the second case it is assumed that all the reflected energy is scattered away without propagating. This scattering problem is discussed in the next section. As another test of the numerical method, Figure 5 shows the effects of thickness on the reflection coefficient for a case studied by Lengyel. The numerical model shows the shift in the reflection null evident in Lengyel's data for plates of 0.313 cm thickness.

## Scattering losses

The parallel plate theory assumes that there are an infinite number of plates while in practice there are only a few (5-6 in the case of Haystack) plates across the beam aperture. With a large number of plates there is little scattering of the reflected waves in directions other than in the direction of the main reflected beam. The scattering results from the "diffraction pattern" of the grating formed by the radiation from the evanescent mode currents on the plates.

The effect of scattering is assumed to subtract energy from the main beam of the reflections and thereby in the limit reduce their forward scatter to zero so that all the reflection energy is removed from the transmitted signal. For example, if the voltage reflection is 10% for each interface then each interface will lose 1% in power for a total of 2%. Table 2 gives the results of calculations of the percentage of scatted energy as a function of the number of plates in the beam.

| # Plates | Scatter % |
|---|---|
| 3 | 90 |
| 5 | 75 |
| 7 | 60 |
| 11 | 30 |
| 15 | 10 |

Table 2. Frequency = 86 GHz; $d$ = 3.2 *mm*
Percentage of reflected power scattered

Non-uniformity in plate separation

Variations in plate separation produces differences in the phase path for each section of the polarizer. A 5% separation change makes a 10° phase error and a 3% power loss.

Summary of losses

For the 86 GHz polarizer design the losses are estimated to be as follows:

| | |
|---|---|
| 1] Ohmic loss | 1 % |
| 2] Reflection losses (with 1 plate) | 0.5% |
| 3] Non-uniformity in plate separation (5% variation) | 3 % |
| Total loss | 4.5% |

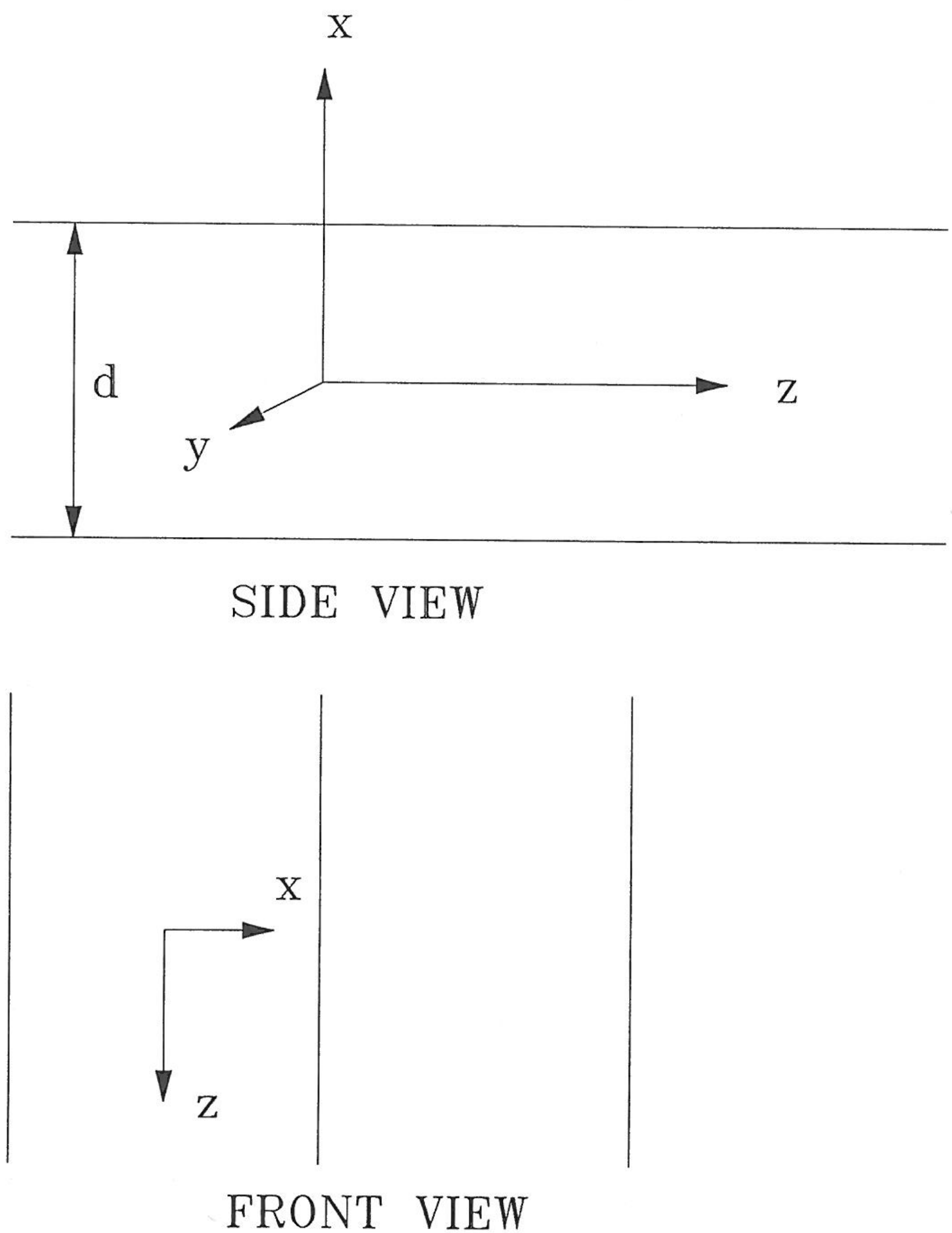


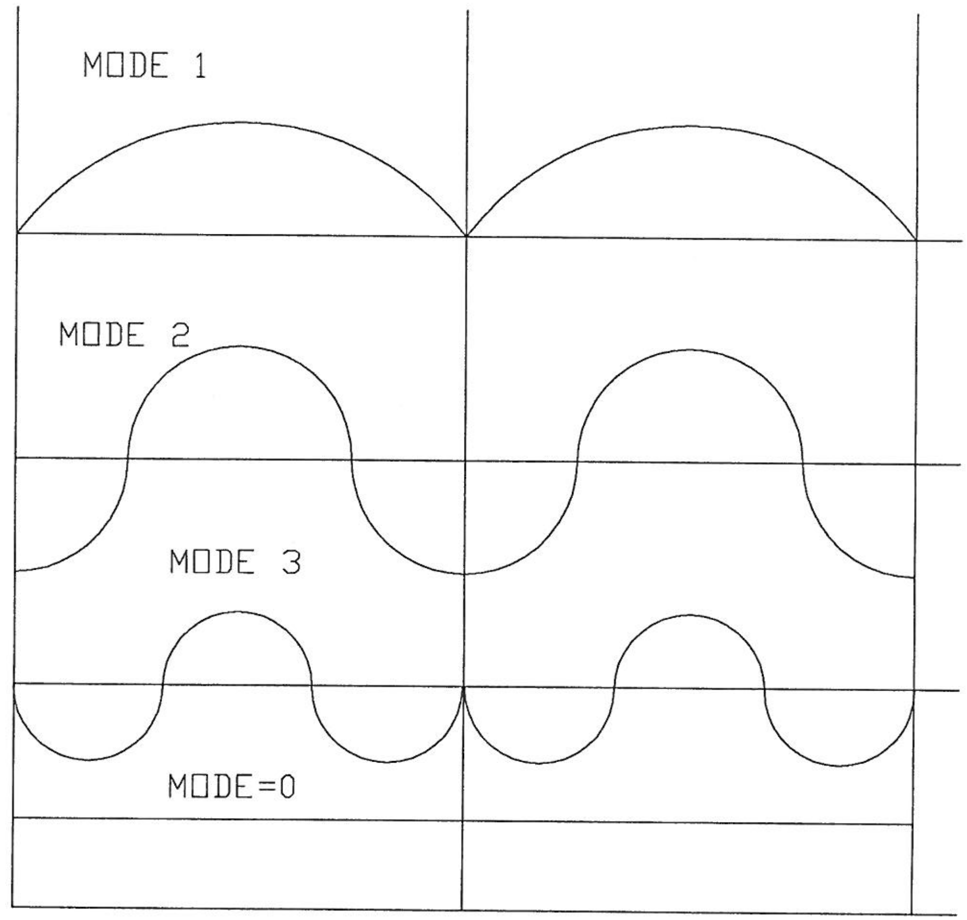

Figure 1 Polarizer Geometry

Primary mode m=1 - only propagating mode between plates. E field goes to zero at walls

Non- propagating mode cannot exist between plates - because it fails to meet boundary condition of zero field at plate walls. This mode can exist in space outside plates

Mode=3 non-propagating mode - evansecent mode which can exist between plates

Mode=0 - plane wave in space cannot exist beteen plates




# Figure 2 Illustration of modes

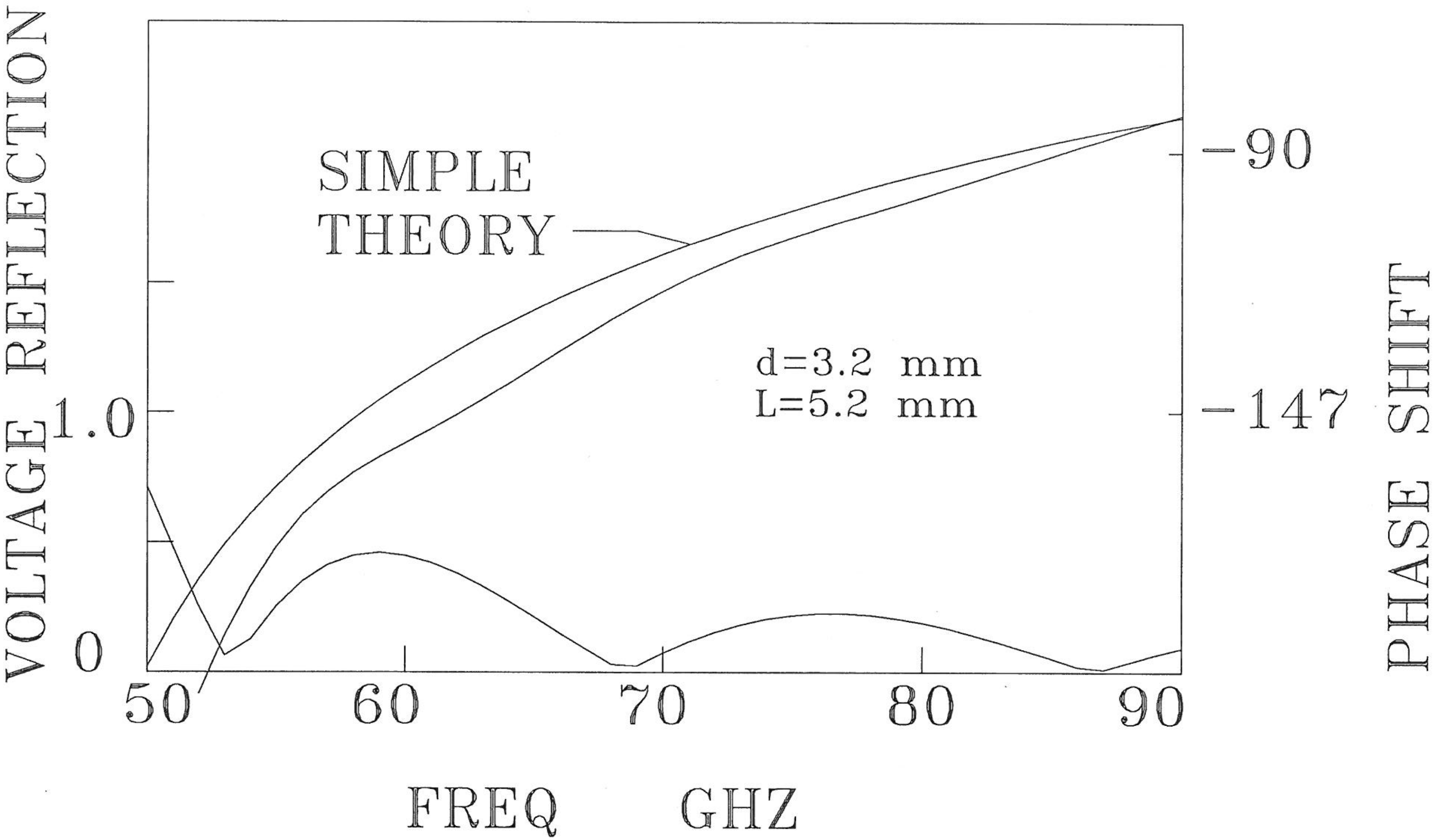



FIGURE 3 POLARIZER PHASE SHIFT AND LOSS USING NUMERICAL SOLN.

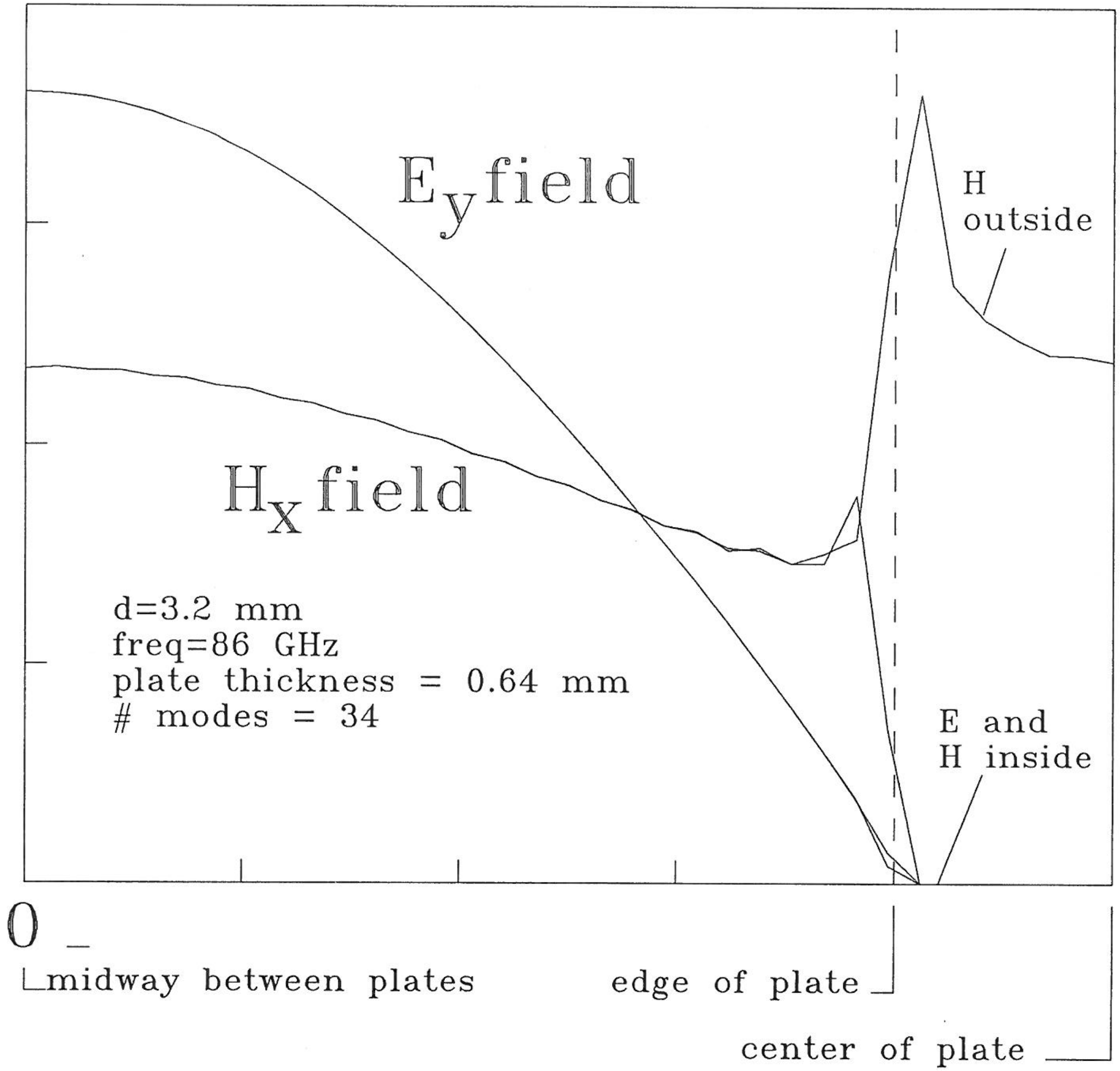
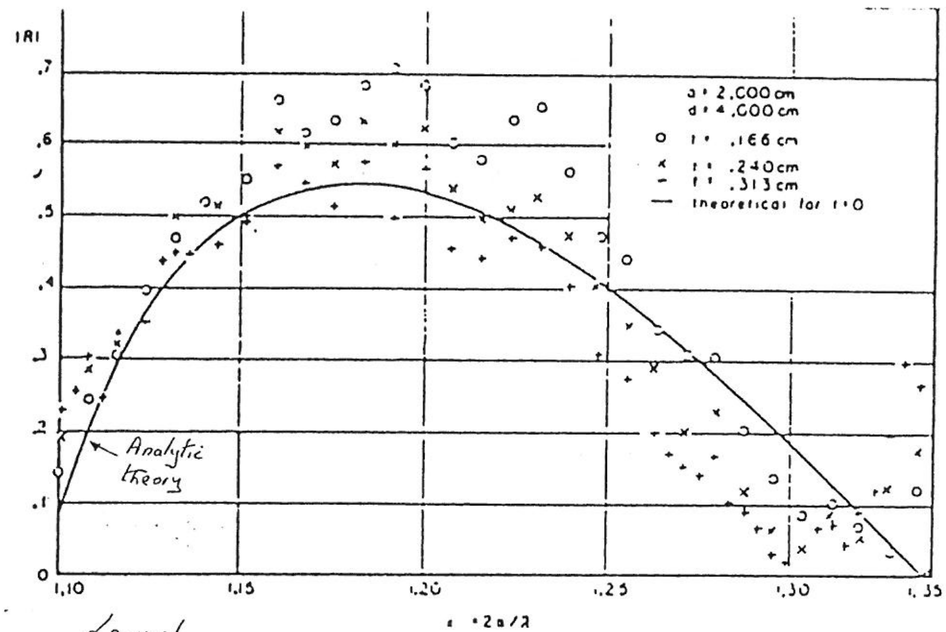



Figure 4 E and H fields




Lengyel

FIG. 11. The effect of plate thickness on the magnitude of the reflection coefficient.

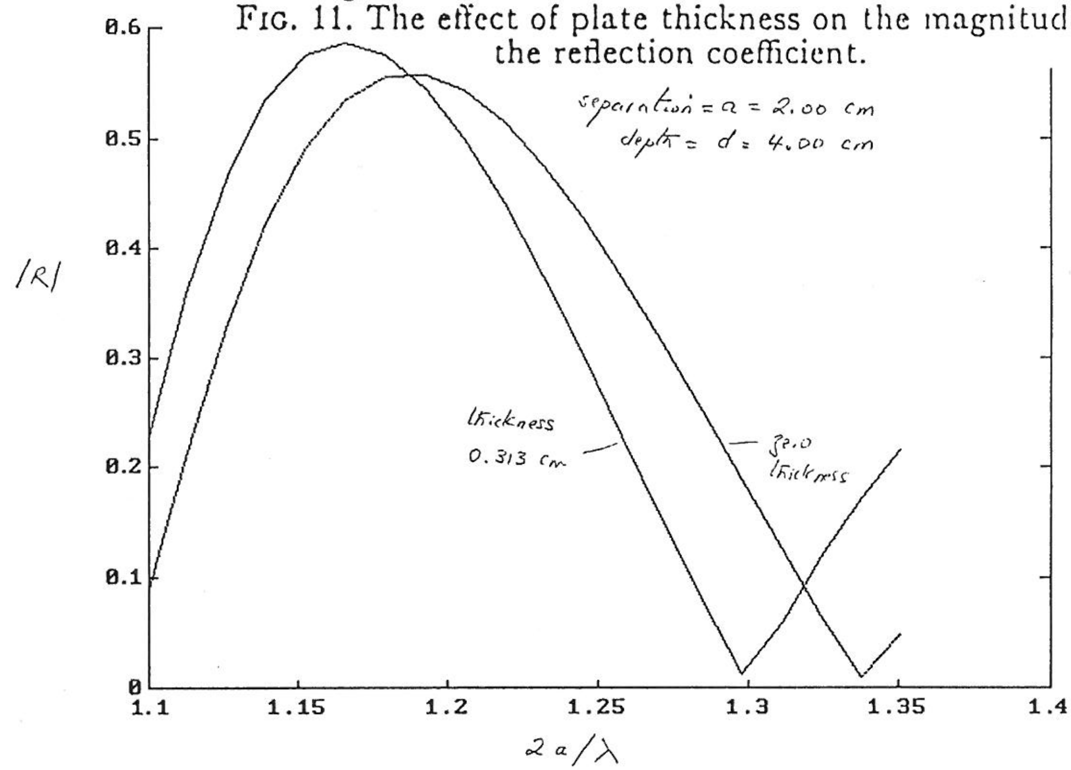



Figure 5.