RFI MEMO #005

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To: RFI Group

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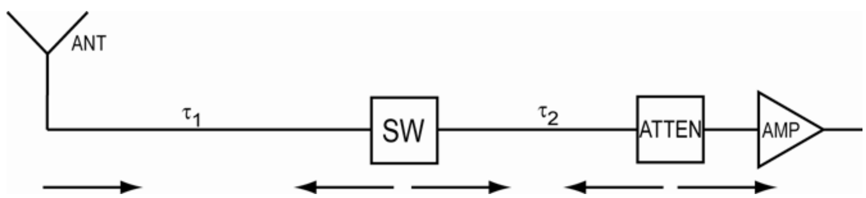
Subject: Effects of reflections in absolute radiometry

The goal of the RFI monitor is to be able to measure the added noise from man's activities. If an ideal antenna and receiver measures the antenna temperature with a beam that illuminates a lossy earth we should measure the ambient temperature of the earth. If we measure more than the ambient temperature we will attribute the added noise to a man made noise.



Consider a model (illustrated in Figure 1) of antenna followed by a cable of delay τ_1 into a switch with loss followed by another cable of length τ_2 into an attenuator into a perfect amplifier. The voltage seen by the perfect amplifier is given by

$$
\begin{aligned}
v = {} & \Gamma T_2^{1/2}(1-b)^{1/2} a b^{1/2} e^{iw(2\tau_1+2\tau_2)} e^{i\phi} \\
& + \Gamma T_1^{1/2}(1-a)^{1/2} a^{1/2} b^{1/2} e^{iw(2\tau_1+\tau_2)} e^{i\phi} \\
& + T_{11}^{1/2}(1-a)^{1/2} b^{1/2} e^{iw\tau_2} \\
& + T_{22}^{1/2}(1-b)^{1/2} \\
& + \left(1-|\Gamma|^2\right)^{1/2} T_{ant}^{1/2} a^{1/2} b^{1/2}
\end{aligned}
$$

where Γ = magnitude of voltage reflection coefficient of the antenna
ϕ = phase of Γ
b = attenuator power loss factor
a = switch power loss factor
$T_2(1-b)$ = noise power emitted by attenuator towards antenna
$T_{22}(1-b)$ = noise power emitted by attenuator towards amplifier
$T_1(1-a)$ = noise power emitted by switch towards antenna
$T_{11}(1-a)$ = noise power emitted by switch towards the amplifier

I assume no cable loss and no reflections at the switch. An attenuator with power loss factor L at ambient temperature T_a emits noise power from both ports equal to $T_a(1\text{-}L)$.

The thermodynamic arguments used to derive the noise emitted from each port of an attenuator can also be used to show that the noise from the 2 ports is uncorrelated when both ports are matched.

The power at the amplifier is given by

$$vv^* \approx 2\Gamma T_a (1-b) ab^{1/2} \cos\left((2\tau_1 + 2\tau_2) + \phi\right) \rho_b$$
$$+ T_{ant} ab$$

dropping terms in $|\Gamma|^2$ and those which are uncorrelated. If we assume $\rho_b \approx 1$ we apply a constant correlation for the amplifier and normalize

$$vv^* \approx T_{ant} + 2\Gamma T_a \left[(1-b) b^{-1/2} \cos\left(w(2\tau_1 + 2\tau_2) + \phi\right) \right]$$

For the $\tau_2 = 0, T_{ant} \approx T_a$ the fraction ripple is $2\Gamma \cos(2w\tau_1 + \phi)\left((1-b)b^{-1/2}\right)$. For a = b = 0.5 the peak to peak ripple is approximately 850 Γ degrees K. If the switch loss is 2 dB and a 1 dB noise figure amplifier is used the peak to peak ripple is reduced to 250 Γ degrees K. Given that it will be difficult to obtain an antenna with Γ much less than 0.3 the ripple is expected to remain at about 75 K peak to peak or a maximum temperature error of about 40 K. Placing the amplifier and switch right at the antenna doesn't really help, as it only increases the ripple the ripple period and in addition the reflection phase is a function of frequency.

In summary this theoretical study suggests that we need a low noise amplifier and a very well matched antenna. In theory we could meet our accuracy goal of 5 K with an amplifier of N.F. under 0.5 dB, and antenna VSWR better than 1.2:1.

Another possibility under consideration is to inject a calibration signal in both directions, towards the antenna and towards the LNA. In the case of a very compact arrangement with the calibration injection between the amplifier and the switch:

$$P_{ant} = T_{ant} + T_R + 2\Gamma T_a \alpha \cos(2w\tau + \phi)$$
$$P_{load} = T_R + T_{load}$$
$$P_{load+cal} = T_R + T_{cal} + T_{load}$$
$$P_{ant+cal} = T_{ant} + T_R + 2\Gamma (T_a + T_{cal}) \alpha \cos(2w\tau + \phi) + T_{cal}$$

where $\alpha \approx (1-b) b^{1/2}$

$$(P_{ant} - P_{load}) = T_{ant} + 2\Gamma T_a \alpha \cos(2w\tau + \phi) - T_{load}$$
$$(P_{load+cal} - P_{load}) = T_{cal}$$
$$(P_{ant+cal} - P_{ant}) = 2\Gamma T_{cal} \alpha \cos(2w\tau + \phi) + T_{cal}$$
$$T_{ant} = (P_{ant} - P_{load}) - (P_{ant+cal} - P_{ant}) T_a / (P_{load+cal} - P_{load}) + T_{load} - T_a$$

These equations assume that the bidirectional noise injection is very close to the amplifier.