MASSACHUSETTS INSTITUTE OF TECHNOLOGY
HAYSTACK OBSERVATORY
WESTFORD, MASSACHUSETTS 01886
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Telephone: 781-981-5407
Fax: 781-981-0590

To: RFI Group
From: Alan E.E. Rogers

Subject: Proposed filter for detection of EOR spectral ramp.

The difficulty in detecting the "global step" in the cosmic background is that there is no obvious way of "Dicke" switching with a comparison. However a true step would be easy to detect as one could simply look for a difference in total power just above step and the power just below the step. Since the global step is expected to be more of a ramp extending over 10 to 20 MHz it will be hard to detect but it may be possible to detect this signature by subtracting a least square polynomial fit to the spectra:

$$z(f) = \left\langle s(f) - \sum_{0}^{N-1} a_k f^{k-N/2} \right\rangle_t$$

where

$s(f)$ = the total power spectrum

a_k = polynomial coefficients which fit the spectrum is a least squares sense.

f = frequency

N = number of polynomial coefficients

$\langle \ \rangle_t$ = average over time blocks

The best fit polynomial coefficients can be found using matrix methods.

$$\left(AA^T\right)^{-1} A^T y$$

where   A = design matrix

y = data vector of $s(f)$

For a frequency range 100 to 230 MHz only about 9 coefficients are needed to fit the expected Galactic spectrum plus receiver noise to within 0.2 mK. The error in the fit is unaffected by any corruption of the spectrum which can be fit by a polynomial with 9 coefficients. On the other hand, add a spectrum with a 20 mK ramp from 120 to 130 MHz and this ramp will become evident in the residual to the fit as shown in the Figure, albeit at a lower amplitude, because It requires many more coefficients for a good fit to the spectrum with the ramp. It should be emphasized, as is discussed in memo 15, that any instrumental effects which vary rapidly with frequency could mimic the signature of the EOR.

The polynomial will perfectly fit any function that is a sum of power law terms

$$f^{k-N/2}$$

for a range of k from 0 to N-1. Non integer exponents are not perfectly fit. The following table gives the peak error in fitting $1000(f/100)^{\alpha}$ for a range of exponent values and number of coefficients.

Index α	Peak error (mK)	N
-3.0	0	5
-2.5	20	5
-2.5	3	7
-2.5	0.2	9
-2.0	0	5
-1.5	12	5
-1.5	2	7
-1.5	0.7	9

EOR

signature after subtraction of polynomial fit to 1000(f/100)⁻²·² K+ 3K + portion of sinusoid which represents a 20 mK EOR ramp

diff (mK)

Frequency (MHz)