UVLBI MEMO #014

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Subject: Detection threshold for bispectral fringe searches

In searching for weak fringes using the bispectrum it is advantageous to use a 2 step process unless the closure phase is known in advance. In our paper (Rogers, Doeleman & Moran, A.J. 109, 1391, 1995) we give the algorithm in equ (74) but do not discuss the detection threshold.
If the known the optimum search algorithm is to search for a maximum in the real part of

$$\sum_{i=1}^{M} amp_i \cos\left(\theta_{ci} - \theta_c\right)$$

where amp_i is the magnitude of the triple product for the ith segment and θ_c is the assumed closure phase. If the closure phase is unknown the search can be made using a 2 step process:

First determine the closure phase by computing

$$R = \sum amp_i \cos\left(\theta_{ci}\right)$$
$$I = \sum amp_i \sin\left(\theta_{ci}\right)$$
$$\theta_c = a\tan 2\left(I, R\right)$$
$$\sum amp_i \cos\left(\theta_{ci} - \theta_c\right)$$

For a single segment this equivalent to finding the maximum magnitude which, as discussed in Rogers et al. degrades the SNR by $\sqrt{2}$. However for a large number of segments there is only a small degradation in SNR over the case of assuming an apriori value for θ_c and in the case of a large number of segments the SNR is approximated by

$$SNR = \frac{Eamp_i \cos\left(\theta_{ci} - \theta_c\right)}{\left(amp_i^2 \sin^2\left(\theta_{ci} - \theta_i\right)\right)^{1/2}}$$

simulations made with

$$s_1 = 3$$
$$s_2 = 1$$
$$s_3 = 0.75$$

#segments	s_3	SNR_1	SNR_2	SNR_3	SNR_4	SNR_5	R1
150	0.75	4.37	4.37	4.97	4.53	5.47	0.94
75	0.75	3.09	3.09	4.30	3.31	5.34	0.86
150	1.0	5.48	5.48	4.97	5.62	5.47	0.93
75	1.0	3.87	3.87	4.30	4.07	5.34	0.85
150	1.5	7.11	7.11	4.97	7.24	5.47	0.93
75	1.5	5.03	5.03	4.30	5.21	5.34	0.83

Where SNR_1 = calculated SNR using equ (49)
SNR_2 = using equ (74) assuming an apriori closure phase
SNR_3 = max SNR_2 in search of 4×10^4 trialswithout signal
SNR_4 = using equ (74) estimated closure
SNR_5 = max SNR_2

The ratio $R = \left(SNR_3/SNR_5\right)/\left(SNR_4/SNR_2\right)$ is a measurement of the equivalent loss of sensitivity for low level detections which results from having to determine the closure phase from the data itself. The loss is in the range of 5 to 16 percent depending on the segment SNRs and the number of segments.