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To: UVLBI Group

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Subject: Effect of coherence on the bispectral averages

Averaging the bispectrum is a sensitive method of obtaining the closure phase and searching for fringes on a weak third baseline in coherence limited VLBI data. Equation (49) of Rogers, Doeleman and Moran (A.J. 109, pp 1391-1401, 1995) gives the segmented bispectral average as

$$SNR_{-b} \approx \frac{s_1 s_2 s_3 \sqrt{M}}{\left(4 + s_1^2 s_2^2 + s_1^2 s_3^2 + s_2^2 s_3^2 + 2s_1^2 + 2s_2^2 + 2s_3^2\right)^{1/2}}$$

where s_1, s_2, s_3 are the segment SNR_2 for the 3-baselines

M = the number of segments

The segment SNRs can be calculated from

SNR = $SNR = L\rho\left(2BT\right)^{1/2}$

Where L = digital loss factor

ρ = correlation coefficient

B = effective bandwidth

T = coherent integration time of each segment

Rogers et al. omits any discussion of the effects of coherence and assumes perfect coherence during the time interval of each segment. In practice it is advantageous to find the time interval, T, that maximizes the SNR of the bispectral average.

If 2 of the three baselines have strong fringes (i.e. $s_1 \gg 1$ *and* $s_2 \gg 1$) then

$$SNR_{-b} \approx s_3 \left(M\right)^{1/2}$$

As long as T is short enough to avoid any coherence loss the SNR will just depend on the total time as

$$SNR_{-b} \approx \frac{s_2 s_3 \sqrt{M}}{\left(2 + s_2^2 + s_3^2\right)}$$

the choices of the optimum segment interval, the coherent integration time T, on each baseline complex because

1] Too long a coherent integration reduces the signal strength so that s^2 becomes

$$s_2 = c_2 L \rho \left(2BT\right)^{1/2}$$

where c_2 is an additional loss factor due to the atmospheric and instrument phase variations.

2] In addition the atmospheric and instrument phase introduce deviations in the closure phase which will be present even at high segment SNR. This reduces the bispectral average SNR by a factor of c_c which depends the coherence loss factors c_1, c_2, c_3. For equal coherence loss on each baseline the results from simulations of a Gaussian walk (the phase rate changes randomly) in each station phase is

c	c_c
1.0	1.0
0.9	1.0
0.8	0.98
0.7	0.93
0.6	0.81
0.5	0.66
0.4	0.50
0.3	0.36
0.2	0.24

For small coherence loss ($c < 0.8$) the closure coherence loss is negligible but becomes significant for large coherence loss.

If there are phase variations at only one site there is no closure coherence loss. Unless the coherence loss is very unequal on the 3 baselines the closure coherence loss factor follows the table above quite closely by substituting the average coherence on the 3 baselines. The results in the table didn't change much with the statistics as long as the spectra of phase variations were dominated by low frequencies which will be the case when the phases are "walking" along with random incremental changes.