MASSACHUSETTS INSTITUTE OF TECHNOLOGY
HAYSTACK OBSERVATORY
WESTFORD, MASSACHUSETTS 01886

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Telephone: 781-981-5407
Fax: 781-981-0590

To: UVLBI Group

From: A.E.E. Rogers

Subject: Maximum likelihood method of detecting periodicity in the closure phase.

For 3 baselines consider the spectrum or complex triple product, b, given by

$b = A_{ab} A_{ac} A_{bc} e^{i(\phi_{ab} - \phi_{ac} + \phi_{bc})}$

where A_{ab}, A_{ac}, A_{bc} are the amplitudes on each baseline and

the closure phase is ϕ_c

$$\phi_c = \phi_{ab} - \phi_{ac} + \phi_{bc}$$

The SNR of each baseline for the coherent integration period is

$$SNR_{ab} = F_{ab} \left(SEFD_a SEFD_b \right)^{-1/2} \left(2BT \right)^{1/2}$$

where F_{ab} is the baseline visibility in Janskys SEFD$_a$, SEFD$_b$ are the system equivalent flux densities in Jankys 2B is the data rate and bits/sec T is the coherent integration time in sec [We ignore quantization loss and other processing loss factors]. The maximum likelihood search for periodicity is made by forming a complex time series from the complex triple product which is then fit by a complex Fourier series of N terms using the method of generalized least squares.

$\hat{s} = \left(A^H w A \right)^{-1} A^H w X$

where X is the vector of the time series complex triple products
w is a optional weight matrix
A is the complex steering matrix
$\hat{s}$ is the signal vector fit to the triple products

For a trial period, per

$A_{jk} = e^{2\pi i t(k) j/per}$

where j is the harmonic index
k is the time index
t(k) is the time of the kth triple product

In practice for ease of C programming real functions are used the steering matrix is calculated using real fitting function of alternating sine and cosine terms. A signal vector of harmonic coefficients is calculated for trial values of the periodic and the power p, the signal calculated from

$$p(per) = \left|A\hat{s}\right|^2$$

for a range of trial periods. The maximum likelihood estimate of the period is given by the period which maximizes P. While in general the maximum likelihood period estimate requires an infinite number of harmonics the results don't change significantly beyond about 5 and the matrix inversion time grows rapidly with the number of harmonics. It can be shown that finding the maximum power in the signal fit is equivalent to finding the minimum of chi squared.

R (GS/s)	MLM (PE)	Spectrum (PE)	ACF (PE)	SNR
0.1	10^{-3} (10^{-1})	0.5	1	1
0.5	$<10^{-3}$ (4×10^{-3})	4×10^{-2}	1	2
1.0	$<10^{-3}$ (3×10^{-3})	2×10^{-2}	1	3
2	$<10^{-3}$ ($<10^{-3}$)	$<10^{-3}$	1	4
4	$<10^{-3}$ ($<10^{-3}$)	$<10^{-3}$	0.3	6
8	$<10^{-3}$ ($<10^{-3}$)	$<10^{-3}$	5×10^{-2}	9
16	$<10^{-3}$ ($<10^{-3}$)	$<10^{-3}$	5×10^{-3}	13

Table 1
Comparison of the detection of periodic structure in SgrA* model B at 230 GHz using Hawaii, SMTO, and CARMA with assumed SEFD's of 12,300, 11900 and 23100 Jansky's respectively. The duration of the simulated observations was 2 hours. The table compares the Maximum Likelihood Method (MLM) with the power spectrum and folded autocorrelation function (ACF). The probability of false detection (PE) is given for data rates from 50 MS/s to 20 Gs/s from the result of 1000 trials at each rate.

Comments on the methods

1] MLM

a. MLM: This method has the best performance. It also has the advantage that it can be used for equally spaced data and even variable coherence intervals through the use of the weight matrix

b. Normalized MLM using only the closure phase
In practice it may be better to use only the closure –phase since the visibility amplitudes could be subject to periodic variations in the telescopes or atmosphere. In this case the signal vector elements are
$$x_k = \cos\phi_c(k)/a + i\sin\phi_c(k)/b$$
where a and b are the rms variation in $\cos\phi_c(k)$ and $\sin\phi_c(k)$ respectively.
The performance is degraded as indicated by the numbers in parentheses in table 1. The normalization of the sine and cosine is most important in the case of small variation in the closure phase.

2] Spectrum
The spectrum was computed from the real and imaginary parts of the bispectrum. Performance is not as good as the MLM because it only approaches the MLM for a sinusoidal periodicity.

3] ACF

The autocorrelation function was computed using

$$ACF(k) = \frac{1}{n-k}\sum_{i=1}^{n-k}\cos(\phi_i - \phi_{i+k})$$

where ϕ is the closure phase. While this is a simple and efficient method but lacks performance especially in the case that closure phases are small. If there are many periods in the observation the ACF can be folded about the period for some improvement.

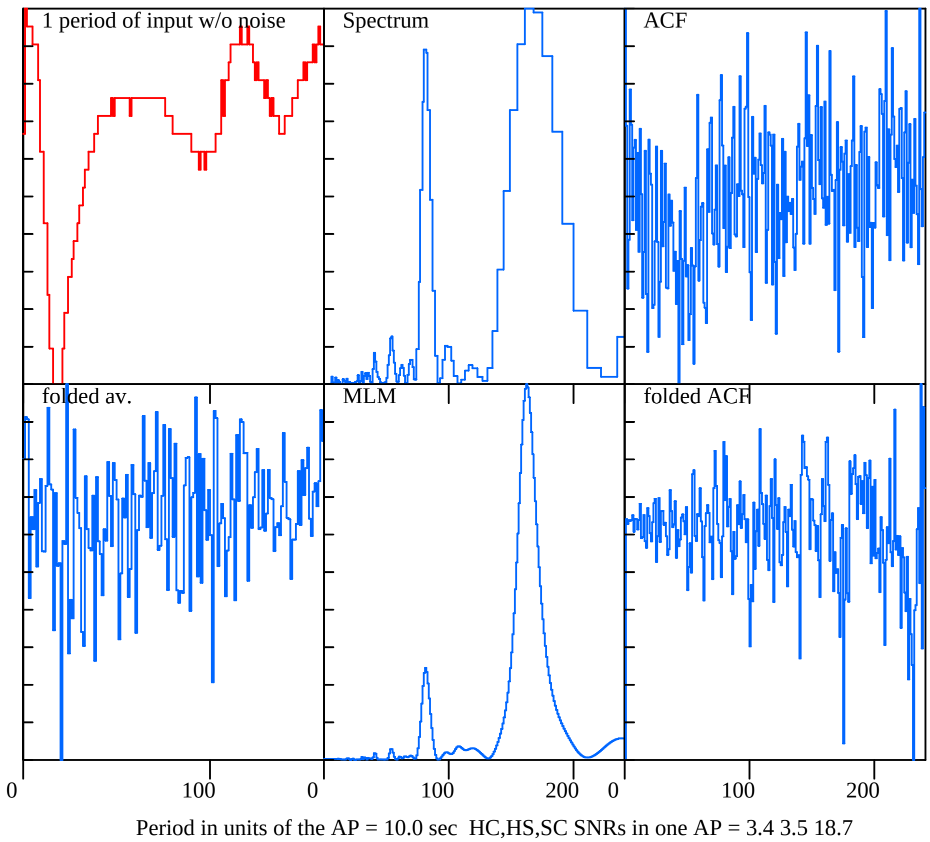

Figure 1. Shows plots of the input wave form, without noise, the spectrum, the ACF, the folded average, the MLM and the folded ACF for a SNR of about 3.

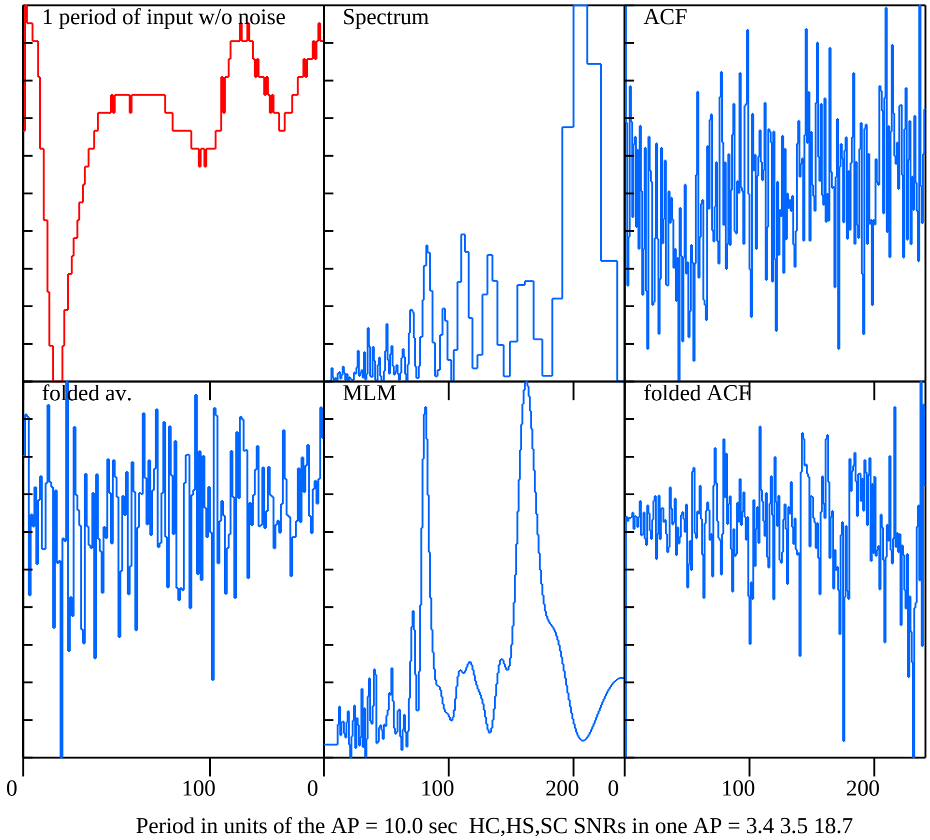

Figure 2. Shows the same plots with MLM and spectrum only using the phase closure information.