VSRT MEMO #020

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To: VSRT Group
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Subject: VSRT fringe detection theory

In the VSRT fringes are detected in the output of a square law detector so that the signal to noise ratio (SNR) for baseline *ab* is given by

$$T_a a^{1/2} b^{1/2} (BT)^{1/2} \big/ \left[(a+b+c)(T_s + T_a) \right]$$

where T_a = antenna temperature of source
T_s = system temperature
B = IF bandwidth (35 MHz)
T = coherent integration (4.267×10^{-4} s)
a,b,c are the power weights of the inputs from antennas a, b and c

For a single baseline system the SNR is maximized with equal power from each antenna and simplifies to

$$SNR = \frac{T_a (BT)^{1/2}}{2(T_s + T_a)}$$

at SNR = 6 for $\left(T_a / (T_a + T_s)\right) = 0.1$

The coherent integration is limited by the coherence of the free running local oscillators in the LNBs to about 1 millisecond. The correlation bandwidth may have to be limited as the fringe amplitude delay function is

$$\left[\sin(\pi B \tau) / (\pi B \tau) \right]$$

where B = I.F. bandwidth
τ = delay difference of signal paths

For example if the full 1 GHz I.F. bandpass is used the correlation will be reduced to zero if the path delay differs by 1 nanosecond so for baselines of about 10 feet the bandpass has to be restricted to well under 100 MHz.

The SNR for amplitude measurements on a single baseline can be improved by incoherent averaging so that

$$SNR = \frac{T_a (BT_c)^{1/2} (T_t / T_a)^{1/4}}{2(T_s + T_a)}$$

and with 1 second the SNR of 6, in the previous example, increases to 42. Unfortunately the PC may not gather data continuously so that a more modest improvement is more likely. For example, with code that is limited to USB 1.0 the factor $\left(T_t / T_a\right)$, which is the number of coherent segments in the total incoherent sum is only 20 in a one second period.

In the case of a 3 element interferometer with reliable fringe detection, within each coherent integration segment, on 2 baselines the data on the 3rd baseline can be coherently processes for arbitrarily long periods. This important advantage is possible through the closure relations which makes the closure phase vector independent of the free running local oscillator phases. That is

$$A_2 = \sum_{K=0}^{K-1} a_{2K}\, e^{i\left(\phi_{0K} - \phi_{1K}\right)} \Big/ K$$

where a_{2K} = complex amplitude of 3rd baseline for the Kth coherent segment

ϕ_{0K} = phase of fringe on 1st baseline

ϕ_{1K} = phase of fringe on 2nd baseline

A_2 = coherent amplitude on 3rd baseline integrated over K segments

References

Rogers, A.E.E., Doeleman, S.S,; Moran, J.M., "Fringe detection methods for very long baseline arrays ," *AJ*, **109**, n 3, March 1995, p 1391-401