VSRT MEMO #021

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To: VSRT Group

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Subject: Calculating the visibility of solar disk

In general visibility of the solar disk is given by a 2-D integral

$$V(z) = \int_0^R \int_0^{2\pi} B(r,\theta) e^{irz(\theta)} r dr d\theta$$

where $B(r,\theta)$ is the brightness as a function of the polar coordinates r and θ. The visibility is normalized by dividing by

$$\int_0^R \int_0^{2\pi} B(r,\theta)\, r dr d\theta$$

The calculation of the 2-D integral can also be done in rectangular *x*,*y* coordinates. In this case the visibility can be rewritten as the 2-D Fourier transform of the brightness. However, to obtain high accuracy a heavily oversampled DFT is needed which may not be efficient despite the power of the FFT. In calculations using interpretative languages like Matlab and Python it is convenient to approximate the calculation using the superposition of several 1-D integrals.

For example, the Sun's brightness at 12 GHz is little different from a uniform disk. In this case, the limb brightening can be approximated by adding an outer ring so that the normalized visibility is

$$V(z) = \left[2J_1(Rz)/(Rz) + FJ_0(Rz)\right] \Big/ [1+F]$$

where the symbols are defined in memo #19. F is the fraction of the Sun's radio output in the enhanced brightness of the limb. [If J_1 and J_0 are not available as library function they are easily calculated using a 1-D integral.]

Unfortunately even with the relatively large 4˚ beam of the VSRT a brightness gradient of up to about ± 20% from limb to limb can be introduced by miss-pointing of the dish so that the Sun lies on the edge of the beam.

A linear brightness gradient effects the imaginary part of the visibility function and can be computed from

$$\alpha \int_0^R \left(R^2 - r^2\right)^{1/2} \sin(rz)\, dr \Big/ \left(\pi R^2/4\right)$$

where α is the gradient fraction (i.e. α = 0.2 for 20%). While

$$\int_0^1 \left(1-x^2\right)^{\frac{1}{2}} \cos\left(a\,x\right) d\,x = J_1\left(a\right)\times\left(\pi/\left(2a\right)\right)$$
$$\int_0^1 \left(1-x^2\right)^{\frac{1}{2}} \sin\left(a\,x\right) d\,x = \text{infinite series}$$

can only be represented as an infinite series (see Gradshteyn and Ryzhik tables of integrals 3.752)