MASSACHUSETTS INSTITUTE OF TECHNOLOGY
HAYSTACK OBSERVATORY
WESTFORD, MASSACHUSETTS 01886
November 19, 2013

To: VSRT Group

From: Alan E.E. Rogers

Subject: Simplified version of the VSRT using a TV dongle

The VSRT hardware can be simplified by using a TV dongle to sample the I.F. signal from the power combiner as shown in the block diagram in Figure 1. This eliminates the need for inline amplifiers and the square law detector. The sampled voltage is squared and the spectrum derived by Fourier transform. The sum of the total power from each antenna is obtained from the DC term and the correlation from the "beat" tone in the spectrum.

The waveform s(t) from the TV dongle is

$$s(t) = n_1(t) + n_2(t) + c(t)\left(1 + e^{i\Delta t}e^{i\theta}\right)$$

Where n_1 and n_2 are the uncorrelated noise waveforms from each LNBF. $c(t)$ is the correlated waveform from a point source and Δ is the difference of the LNBF local oscillator frequencies. Note that the dongle outputs a complex waveform. If the LNBFs have equal gain then $s(t)$ can be simplified to

$$s(t) = n(t) + c(t)\left(1 + e^{i\Delta t}e^{i\theta}\right)$$

Where θ is the "fringe" phase. The average signal power is

$$\langle ss*\rangle = \langle nn*\rangle + \langle cc*\rangle\left\langle\left(2 + e^{i\Delta t}e^{i\theta} + e^{-i\Delta t}e^{-i\theta}\right)\right\rangle = \langle nn*\rangle + 2\langle cc*\rangle$$

The "fringe power" is obtained from the "beat tone" in the spectrum of ss*. When the trial LO offset equals the actual LO offset

$$\left\langle ss*e^{-i\Delta t}\right\rangle = \left\langle nn*e^{-i\Delta t}\right\rangle + 2\left\langle cc*e^{-i\Delta t}\right\rangle + e^{i\theta}\langle cc*\rangle$$

The normalized correlation, V, also known as the visibility, is given by the ratio of correlated power, F, to total power, P/2, from each antenna.

Where $V = 2F/P$

$$P = \langle ss*\rangle$$

$$F = \left\langle ss*e^{-i\Delta t}\right\rangle$$

If the standard deviation of each complex component of the LNBF noise is σ_n and that of the correlated noise is σ_c

$$P = 4\sigma_n^2 + 4\sigma_c^2$$

$$F = 2\sigma_c^2$$

To estimate the signal to noise ratio (SNR) in the measurement consider the following relationship for Gaussian statistics

$$\left\langle \left(n^2 - \left\langle n^2 \right\rangle \right)^2 \right\rangle = \left\langle \left\langle n^4 \right\rangle - 2\left\langle n^2 \right\rangle^2 + \left\langle n^2 \right\rangle^2 \right\rangle = \left(3\sigma^4 - 2\sigma^4 + \sigma^4 \right)\big/ N = 2\sigma^4 / N$$

Where n is a real Gaussian random of standard deviation σ. N is the number of independent measurements of n^2. From this relation the value of $|V|$ when no correlated signal is present is given by

$$|V| = \frac{2|F|}{P} = \frac{2 \times 4 \times \left(2\sigma_n^4 / N \right)^{\frac{1}{2}}}{4\sigma_n^2} = \left(8/N \right)^{\frac{1}{2}}$$

The number of independent measurements for the dongle's bandwidth B over a time span T is 2BT so the correlation noise floor is $2BT^{-\frac{1}{2}}$ in units of the visibility. The SNR for the measurement of the magnitude of the visibility is $(1/2)|V|(BT)^{\frac{1}{2}}$. This can be improved by a factor 2 using a separate dongle for each antenna at the expense of added complexity. In practice the integration time, T, is limited by the coherence of the local oscillators. It is possible to link the dongles by feeding the crystal output from one unit into the other. Also it is possible to obtain LNBFs which operate from an external 10 MHz reference to overcome the coherence limitation. However the performance of the simple arrangement of Figure 1 is adequate for most laboratory experiments and for single baseline measurements of the Sun provided LNBFs with relatively low phase noise are used. Typical coherence for original triple feed Direct TV LNBFs is about 10 msec which sets the visibility threshold to 1% for the dongle's 3.5 MHz bandwidth. For fixed coherent integration some further improvement in SNR can be made by incoherent averaging of $|V|^2$. The advantage of averaging $|V|^2$ instead of $|V|$ is that it allows an unbiased estimate of the visibility by subtraction of the noise bias prior to taking the square root. However, the SNR improvement is proportional to $T^{\frac{1}{4}}$. See "Fringe detection methods for very long baseline arrays," Rogers, Doeleman, and Moran *Astron J.* **109**, 3, pp 1391-1401, 1995)

The TV dongle also provides accurate measurements of the power from the sum of 2 LNBFs or a single LNBF which, while not as sensitive a measure as the correlation, is useful for many laboratory experiments and demonstrations. More information on the specific recommendations for LNBFs will become available along with software written in C for use under the Linux operating system.
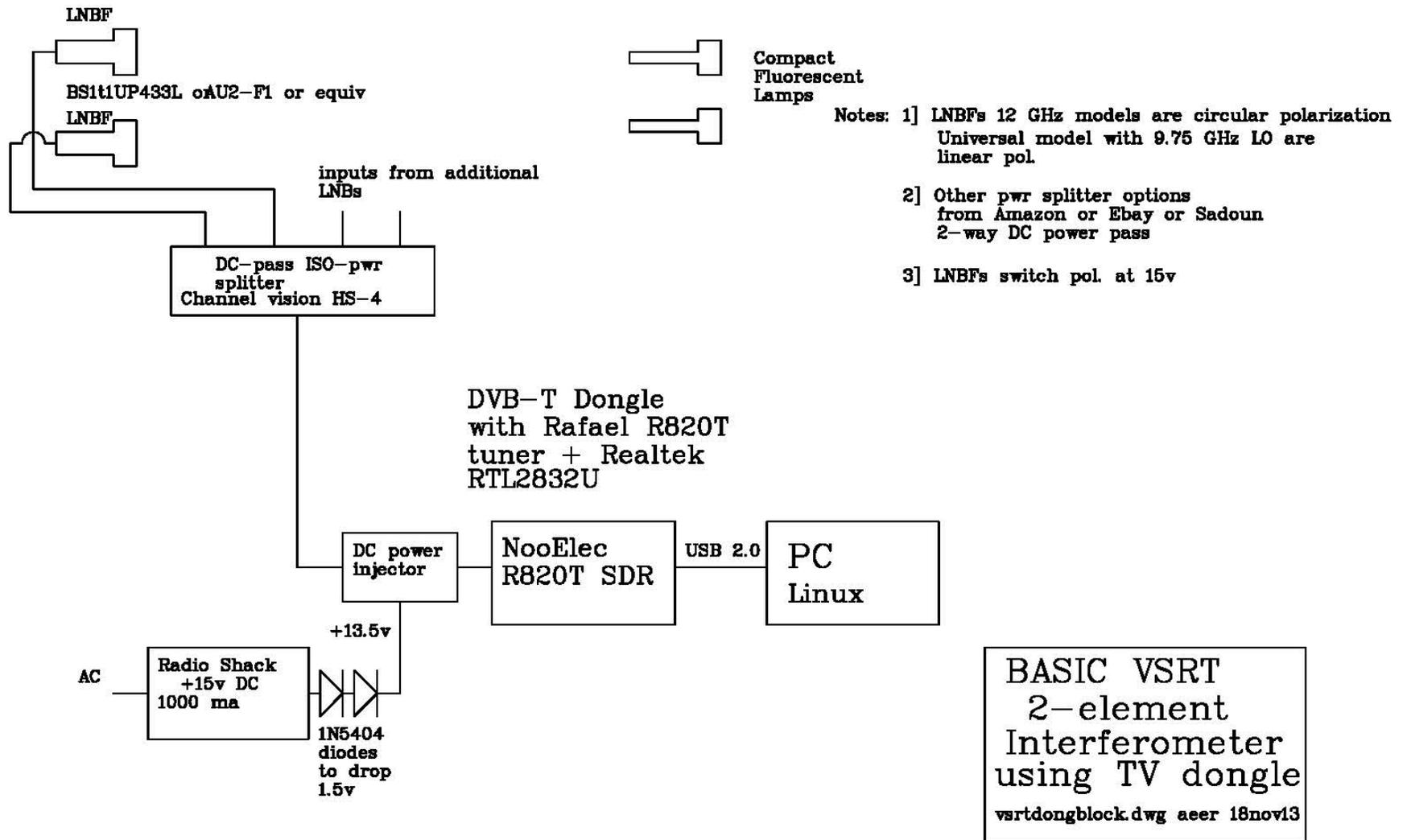
Figure 1. Block diagram of the VSRT