New Developments in Maser Theory

Vladimir Strelnitski
Maria Mitchell Observatory

"Radio Stars"
Haystack Observatory
4 October, 2012

Plan

1. **Summary of Maser Theory**
    - ***Uniqueness of Inversion***
    - ***Pumping Cycles***
    - ***Thermalization***
    - ***Theoretical Pump Power***
    - ***Saturation***
    - ***Observational Requirements for the Pump Power***
2. **Hydrogen Masers and Lasers**
3. **Prospects**

Uniqueness of Inversion

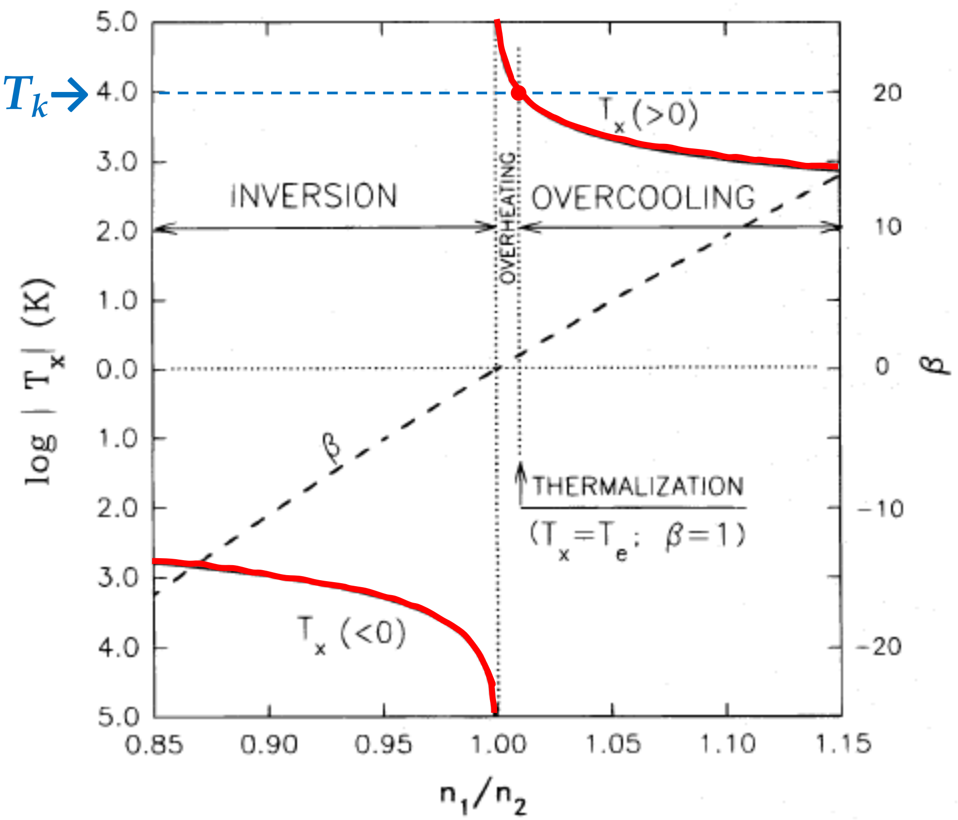
$$T_x = \frac{h\nu}{k}\left[\ln\left(\frac{n_1}{n_2}\right)\right]^{-1}$$

$T_x = T_k$ - Thermalization

$0 < T_x < T_k$ - Overcooling

$T_k < T_x < \infty$ - Overheating

$T_x < 0$ - Inversion



$h\nu/k \ll T_k;\ |T_x|$

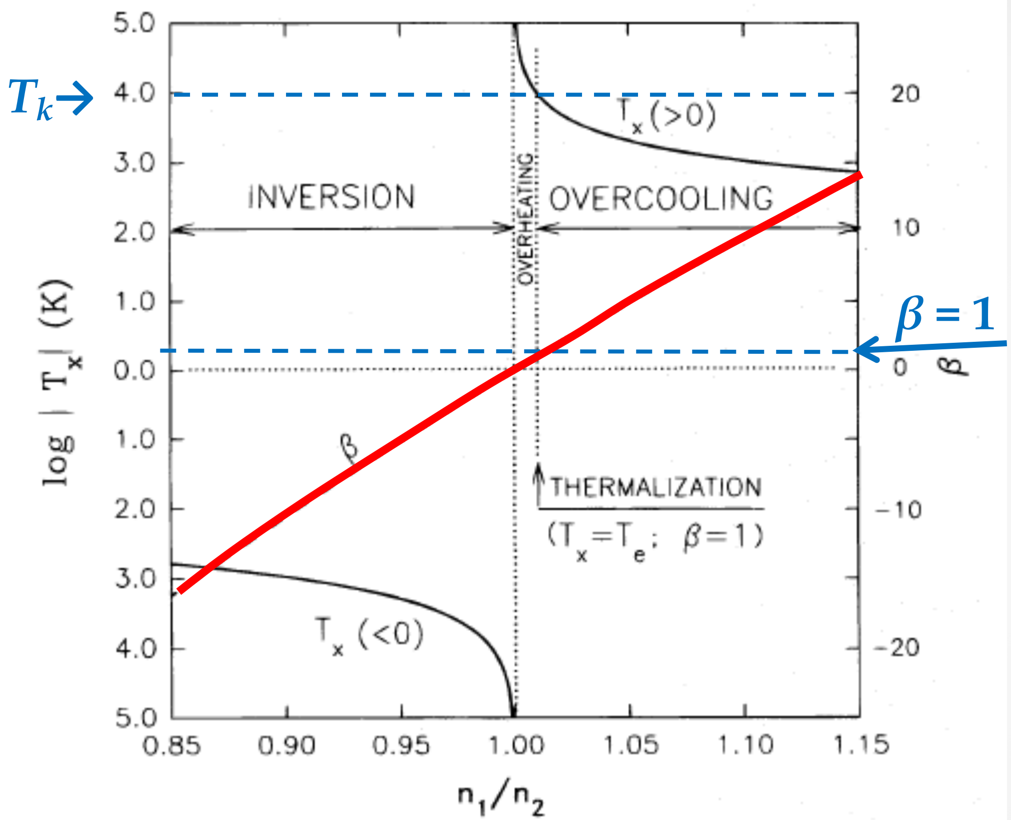
$$\beta_{12} \equiv \frac{1-\exp(-\frac{h\nu}{kT_x})}{1-\exp(-\frac{h\nu}{kT_k})} \approx \frac{T_k}{T_x}$$

$\beta = 1$ - Thermalization

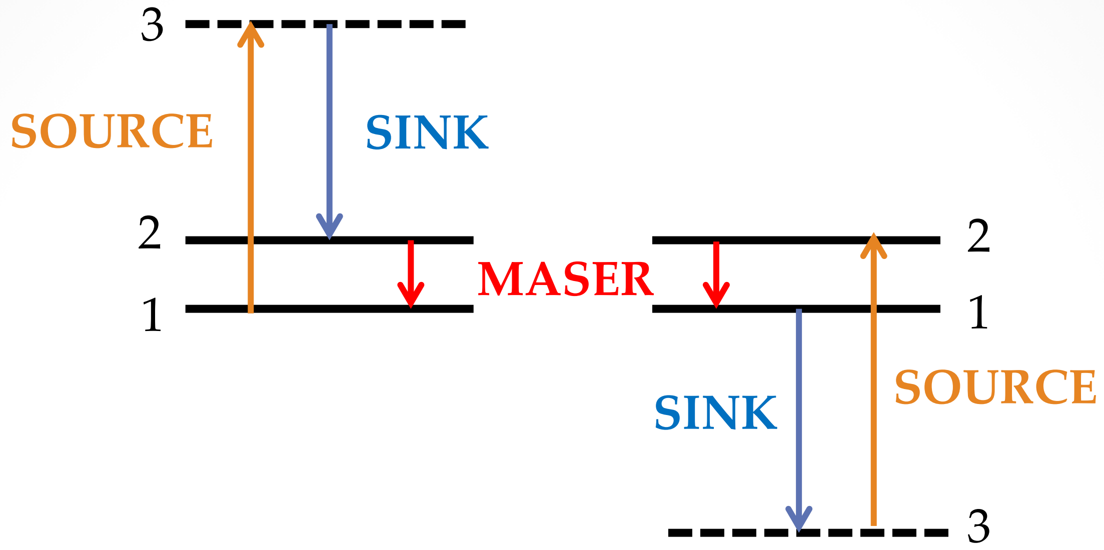
$\beta > 1$ - Overcooling

$0 < \beta < 1$ - Overheating

$\beta < 0$ - Inversion



Pumping Cycles



General Recepe: Look for TWO temperatures!

Energy Reservoirs:

- *radiation (star; dust)*
- *collisions (maxwellized gas; streams)*
- *chemical processes (ionization; dissociation; dust coat sublimation…)*

Deguchi (1977)

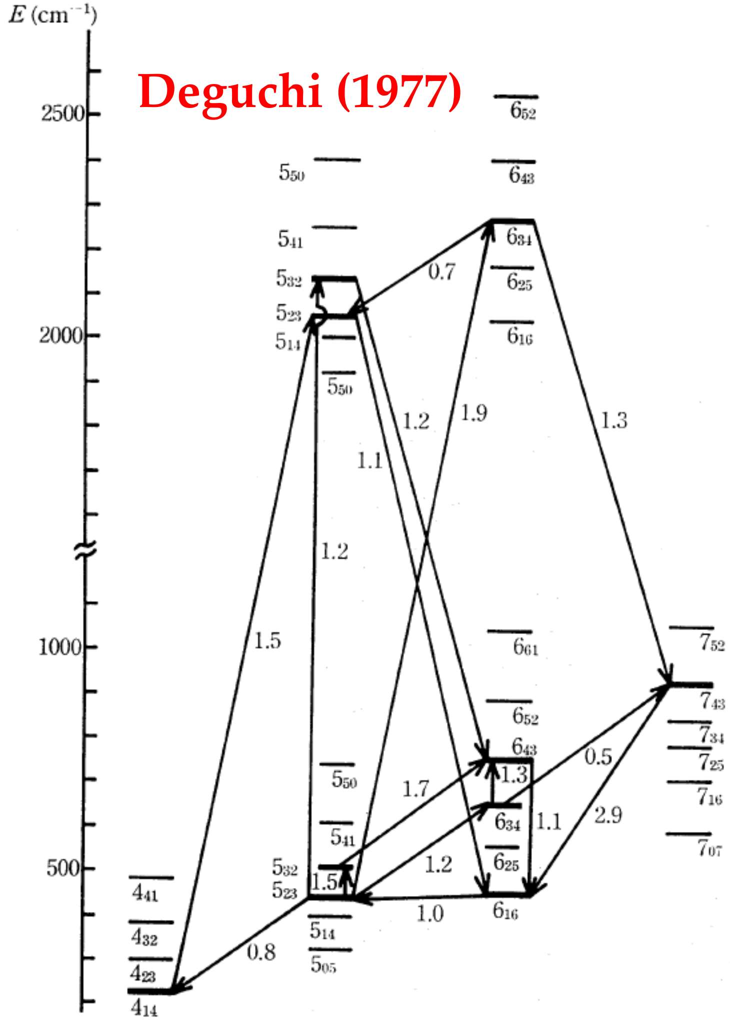


Fig. 2. Pumping cycles of 6_{16}–5_{23} maser of ortho-water at $R=6.9\times10^{13}$ cm for the case of $\dot{M}=3\times10^{-6}M_{\odot}\,\mathrm{yr}^{-1}$, $T_*=2000$ K and $T_K{}^0=T_*/1.4$. The population flow rates are normalized to the 6_{16}–5_{23} maser flow rate. The cycles which do not lead to the population inversion are not shown.

Sobolev & Strelnitski (1983)

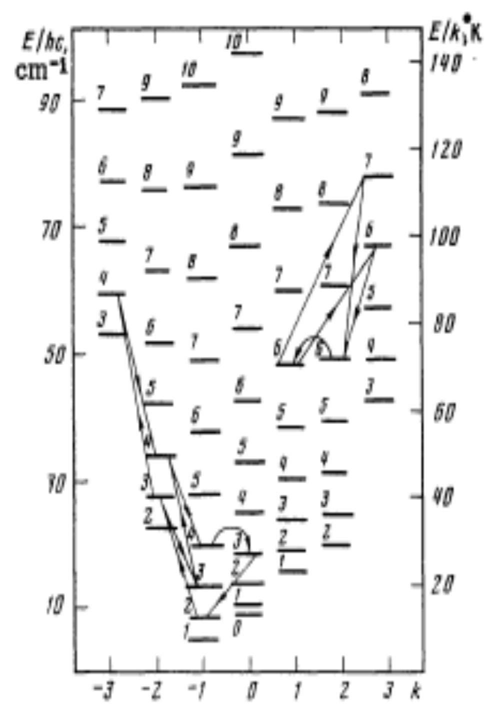


FIG. 2. Typical maser pump cycles by the transitions $4_{-1}\rightarrow3_0$ (for Sgr B2) and $6_2\rightarrow6_1$ (for OMC 1).

Sobolev & Deguchi (1994)

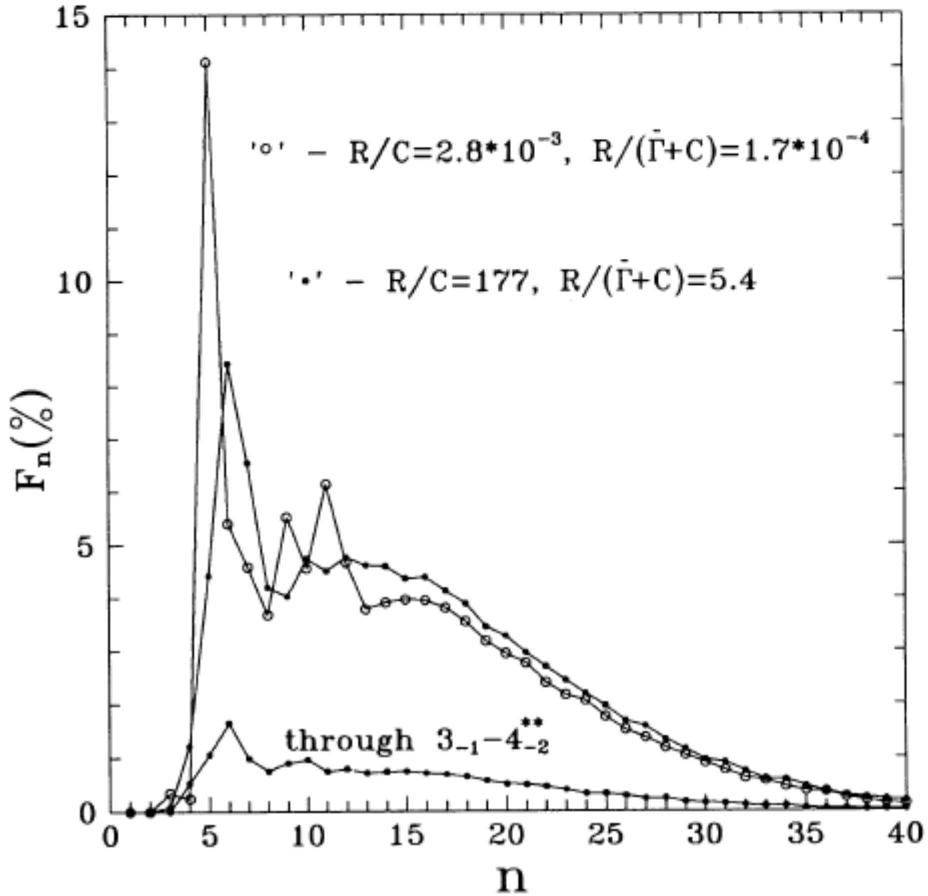


FIG. 2.—Contributions of pump cycles with different numbers of links, *n*, to the maser flux of the 2_0–3_{-1}E transition in Class II methanol masers with different saturation degrees. The curve for low saturation degree is marked by open circles. That for high saturation degree is marked by dots. Contributions of the cycles which pass through the link $3_{-1} \rightarrow 4_{-2}$**E in the highly saturated case are shown separately. The double asterisk marks the level of the second torsionally excited state.

Gray (2007)

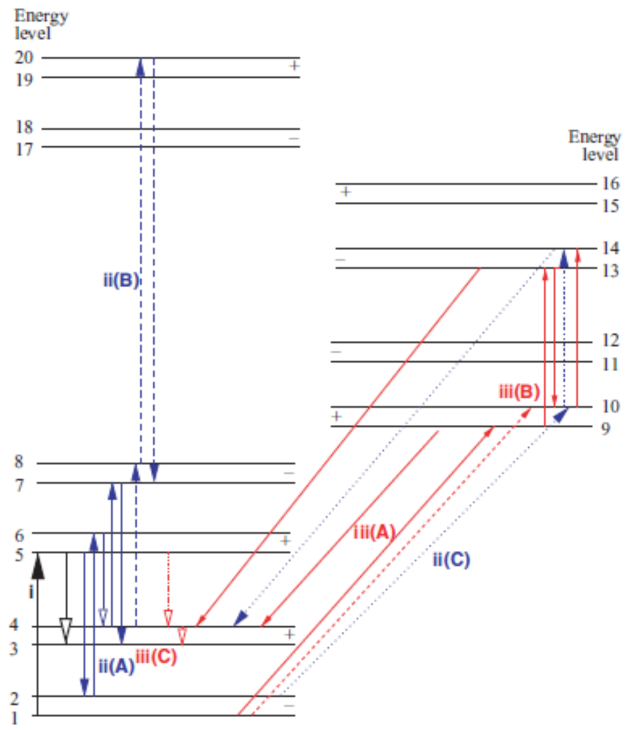
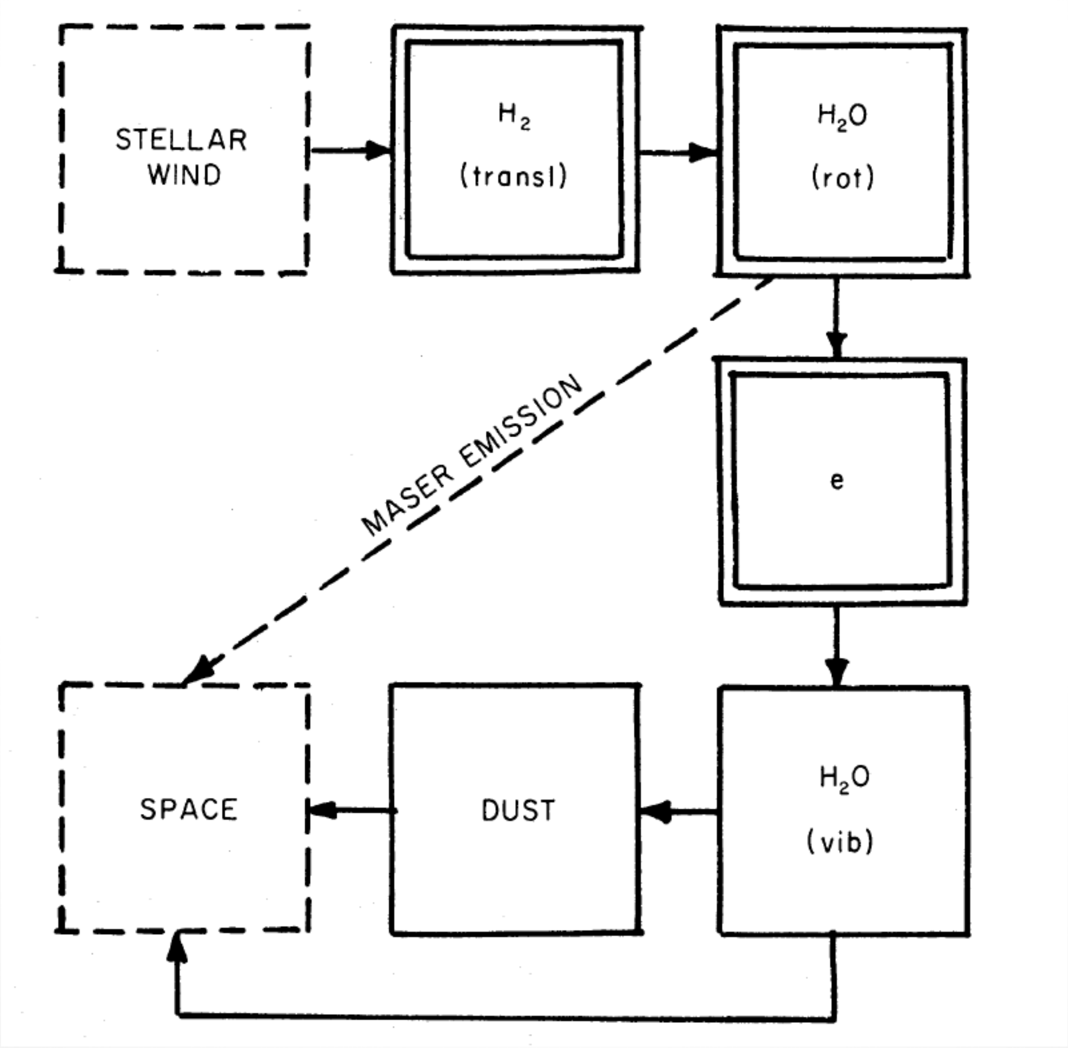


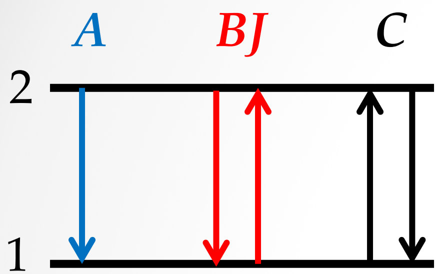
Figure 2. Principal pumping routes for the 1665-MHz maser under the conditions of Table 2. Energy levels are numbered following the scheme in Table 1, and are marked with a parity assignment (+ or −). Energy gaps between levels are not drawn to scale. Route 1 transitions are shown in black; additional transitions appearing in Route 2 are drawn in blue, and further additions from Route 3 are shown in red. Transitions appearing from an 'A' subroute are marked with the solid lines, those from a 'B' subroute are marked with the dashed lines, those from a 'C' subroute are marked with the dotted lines. For simplicity, only the forward (pumping) routes are shown for each antagonistic pair, and transitions appearing in more than one route are drawn only once. A solid arrowhead indicates a transition allowed for electric dipole radiation; a hollow arrowhead indicates a radiatively forbidden transition in which population transfer can be considered to proceed via collisions only.

Energy Drain in CCr Pumping of H20



Thermalization

Two-level System:



$$\frac{N_2}{N_1} = \frac{C_{12} + B_{12}J_{12}}{A_{21} + C_{21} + B_{21}J_{12}}$$

$\tau_{\min} \ll 1$ ⇒

$C \gg A\,;BJ$

$$\frac{N_2}{N_1} \approx \frac{g_2}{g_1}\, e^{-\frac{h\nu}{kT_k}}$$

$C \ll A;\,BJ$

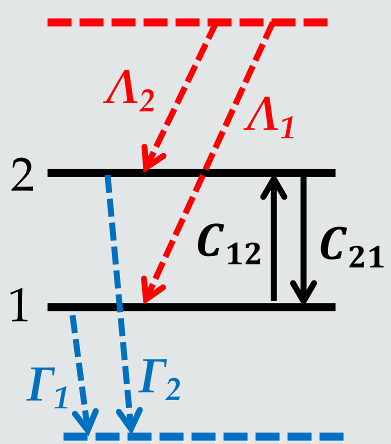
$$\frac{N_2}{N_1} \approx \frac{g_2}{g_1}\, e^{-\frac{h\nu}{kT_r}}$$

$\tau_{\min} \gtrsim 1$ ⇒

(Avrett & Hummer 1965)

$$T_x \to T_k \approx T_r \quad \text{when} \quad C \gtrsim \frac{A}{\tau_{\min}}$$

Maser (unsaturated)



$$\Delta N_0(\mathrm{cm}^{-3}) \approx \frac{\Lambda_2 - \Lambda_1}{\Gamma + C}$$

- CC or XX pumping:
  NO thermalization!

  Strelnitski (1984)

  Norman & Kylafis (1987)
- CR, RC … pumping:
  possible delay of thermalization
  to higher densities (example
  below: Hydrogen masers).

  Strelnitski *et al.* (1996)

Estimates of the Pump Power

$$n_1 \Delta P_{\mathrm{RRv}} \sim \frac{n_1 \exp(-E_{\mathrm{v}}/kT_{\mathrm{v}})\, A_{\mathrm{v}}}{\tau_0^{\mathrm{v}} \sqrt{\pi \ln(\tau_0^{\mathrm{v}}/2)}}$$

RRv:
Litvak (1969)

$$\Delta P_{\mathrm{RCv}} \sim n_{\mathrm{H}} q_{\mathrm{H}}^{\mathrm{v}} [\exp(-E_{\mathrm{v}}/kT_{\mathrm{v}}) - \exp(-E_{\mathrm{v}}/kT_{\mathrm{H}})]$$
$$\lesssim n_{\mathrm{H}} q_{\mathrm{H}}^{\mathrm{v}} [\exp(-E_{\mathrm{v}}/kT_{\mathrm{d}}) - \exp(-E_{\mathrm{v}}/kT_{\mathrm{H}})],$$

RCv:
Goldreich & Kwan (1974)

$$\Delta P_{\mathrm{CCr}} \sim n_{\mathrm{e}} q_{\mathrm{e}}^{\mathrm{r}} \frac{E_{\mathrm{r}}}{kT_{\mathrm{e}}} \left(\frac{T_{\mathrm{H}} - T_{\mathrm{e}}}{T_{\mathrm{H}}} \right)$$

CCr:
Strelnitski (1982)

Saturation

$R \equiv B_{21}J_{21} << \Gamma + C$ $\quad \Delta N_0 \approx \dfrac{\Lambda_2 - \Lambda_1}{\Gamma + C}$ **(Unsaturated inversion)**

→

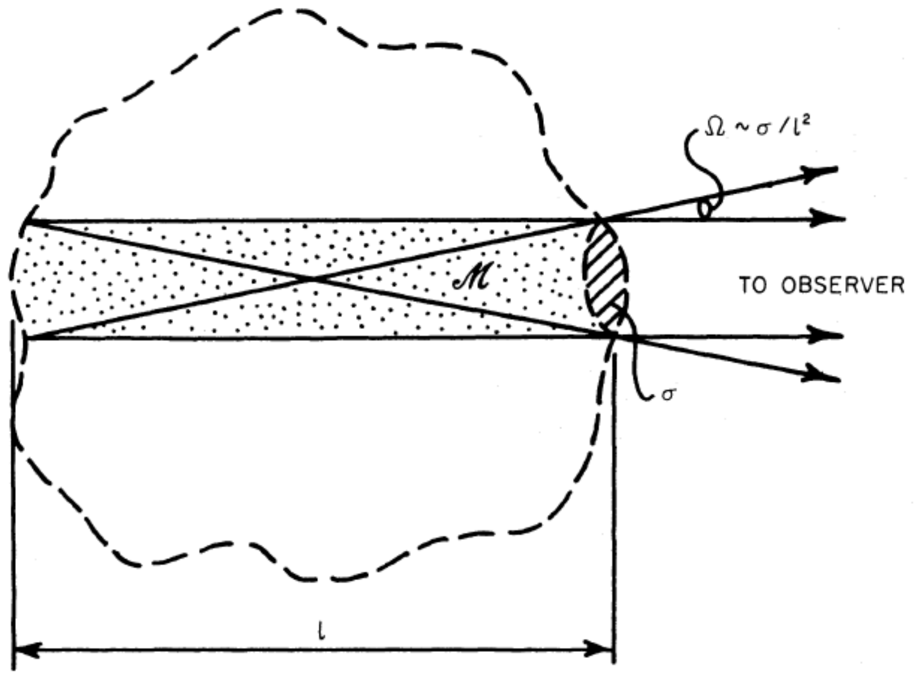
$R \gtrsim \Gamma + C$ → $\Delta N \approx \dfrac{\Delta N_0}{1 + \dfrac{B_{21}J_{12}}{\Gamma + C}} \approx \dfrac{\Delta N_0}{1 + \dfrac{J_{12}}{J_s}}$ **(Saturation)**

$$Js \equiv \frac{\Gamma + C}{B_{21}}$$

Saturation results in:

- Linear intensity growth
- The rule of “1 maser photon per pumping cycle“

Observational Requirements for the Pump Power



$$\left(\frac{n_1}{\mathrm{cm}^{-3}}\right)\left(\frac{\Delta P}{\mathrm{s}^{-1}}\right) \gtrsim 10 \left(\frac{l}{\mathrm{AU}}\right)^{-3}\left(\frac{D}{\mathrm{kpc}}\right)^{2}\left(\frac{F}{\mathrm{Jy}}\right)\left(\frac{\Delta\nu_{12}/\nu_{12}}{10^{-6}}\right)$$

Hydrogen Masers and Lasers

D. Menzel: The Man Who Could Make the Whole Story Happen Earlier

PHYSICAL PROCESSES IN GASEOUS NEBULAE

III. THE BALMER DE...

...CESSES IN GASEOUS NEBULAE

...DIATION

> Outside of thermodynamic equilibrium, the condition may conceivably arise when the value of the integral [*of the light absorption in a line*] turns out to be negative. The physical significance of such a result is that energy is emitted rather than absorbed. This energy must be distinguished, however, from that arising in random emissions. The process merely puts energy back into the original beam, as if the atmosphere had a negative opacity. <u>This extreme will probably never occur in practice.</u>

where it app... The source of the luminosity ... This assumption ... in a few cases, imbedded in the nebula. This assumption ... despite our failure to locate the exciting star in a few cases, NGC 6960, the well-known Network Nebula, where perhaps the star is variable or obscured by cosmic dust.

Fundamental to any discussion of physical processes is the evaluation of the rates of emission and absorption of radiation by atoms in various quantum states. The relation of the radiation field to the transitions that may occur was laid down by Einstein[1] for discrete states and was generalized by Milne[2] to include continuous atomic states. Numerical calculations, however, require a knowledge

...8, 121, 1917. [2] *Phil. Mag.*, 47, 209, 1924; *ibid.*, 547, 1925.

...atoms per cubic centimeter in level n, ... (1)
...Einstein spontaneous probability coefficient, and ν is the ...cy. Outside of thermodynamic equilibrium, N_n must be cal...by balancing the number of atoms leaving a given quantum ...nst those entering, by all possible routes. We thus have an ... of simultaneous equations to solve, one equation for each ...method of solving these equations, for certain specified ...ditions, was given in II. The results are most con-

...llows, Harvard University.

...85, 330; 86, 70, 1937.

Baker & Menzel (1937)

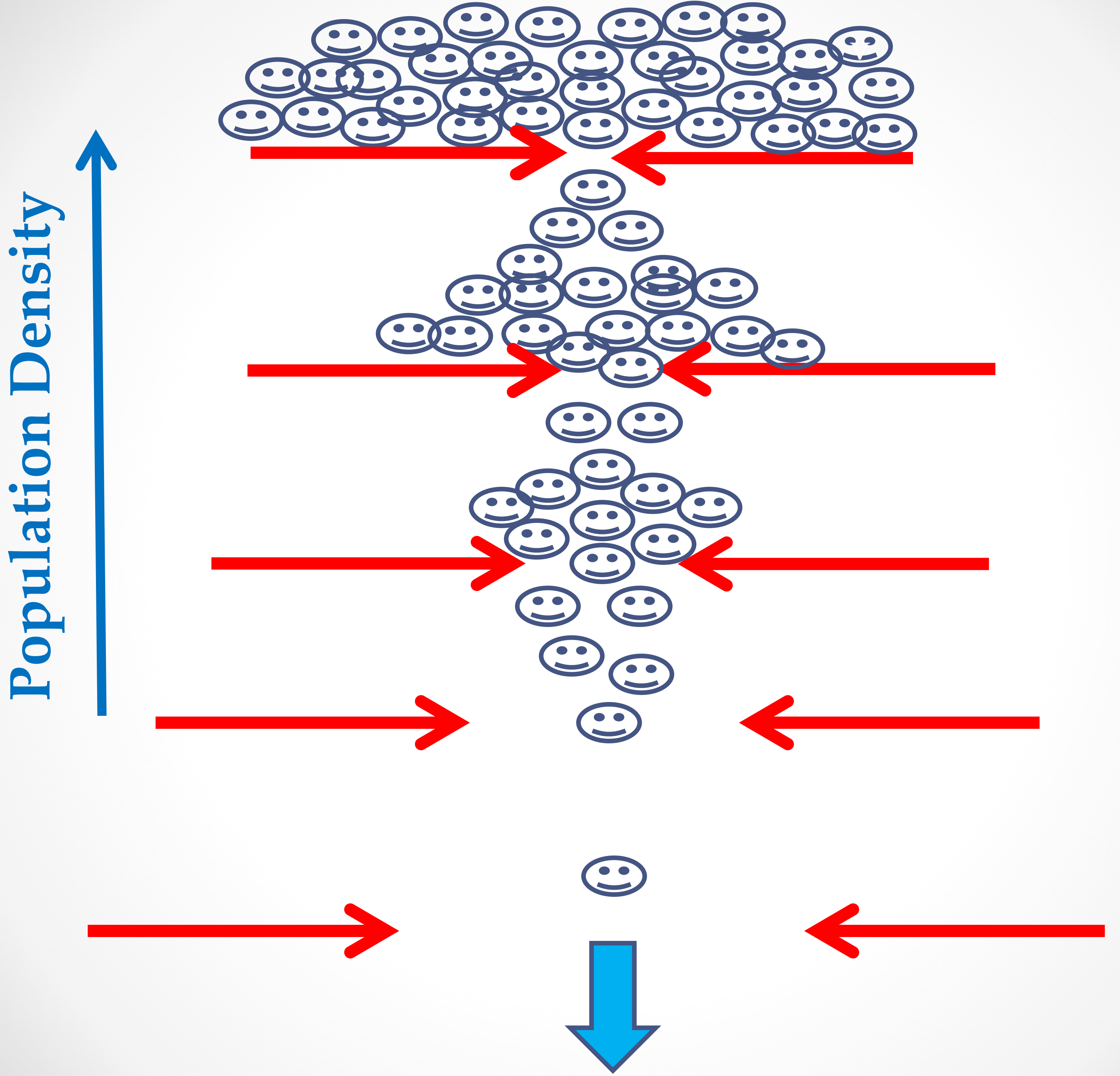
TABLE 6

b_n (CASE B)

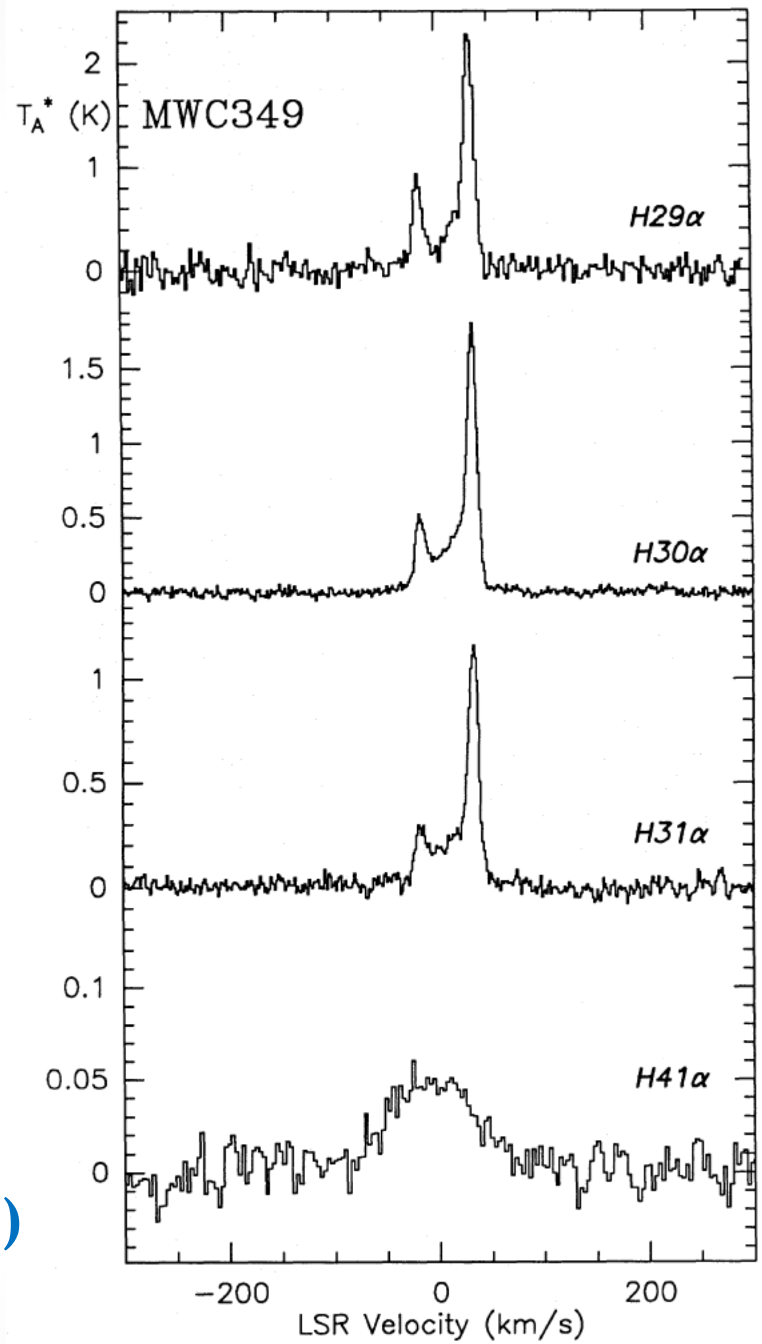
n	T_ϵ 5,000° b_n	5,000° b_n (Cillié)	10,000° b_n	10,000° b_n (Cillié)	20,000° b_n	20,000° b_n (Cillié)	20,000° 14-State b_n	40,000° b_n	80,000° b_n
3......	0.0098	0.0120	0.089	0.111	0.330	0.409	0.420	0.670	1.304
4......	.0406	.0448	.166	.188	.404	.453	.466	.729	1.108
5......	.0840	.0878	.233	.251	.460	.494	.510	.743	1.065
6......	.132	.131	.290	.298	.512	.520	.538	.773	1.045
7......	.173	.166	.341	.333	.550	.538	.558	.789	1.036
8......	.211	.195	.379	.357	.577	.547	.568	.802	1.032
9......	.244	.218	.417	.373	.600	.549	.570	.816	1.031
10......	.271	0.236	.448	0.383	.620	0.545	0.565	.825	1.030
15......	.371		.526		.676			.853	1.027
20......	.428		.571		.713			.871	1.025
25......	.464		.600		.736			.882	1.024
30......	0.486		0.618		0.750			0.893	1.024

All the transitions above *n*=5 are inverted!

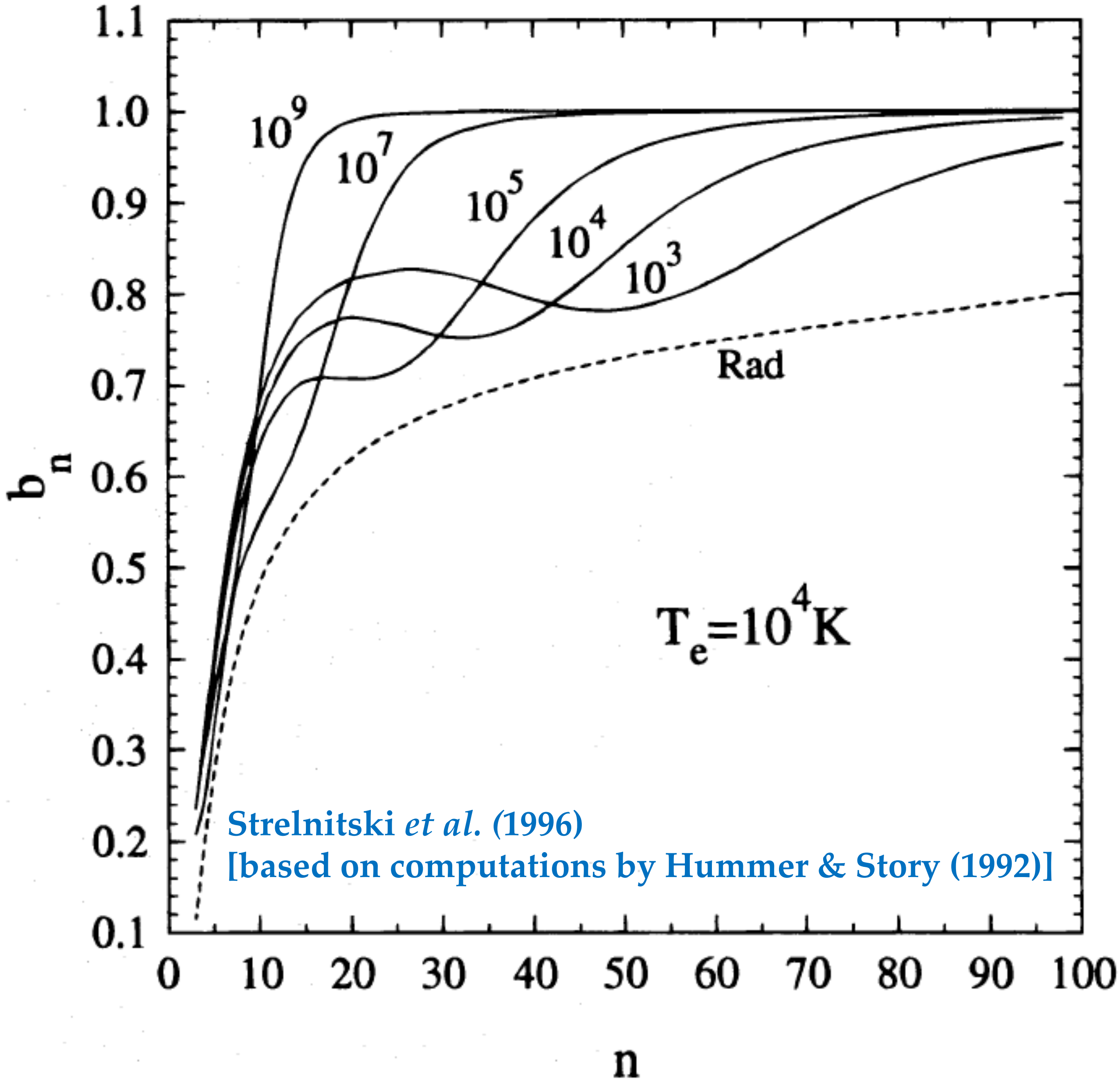
ment is insensitive to temperature. In view of the extreme physical conditions that exist in nebulae, the partition of atoms among the various excited levels approaches surprisingly close to the thermodynamic value.


Population Density

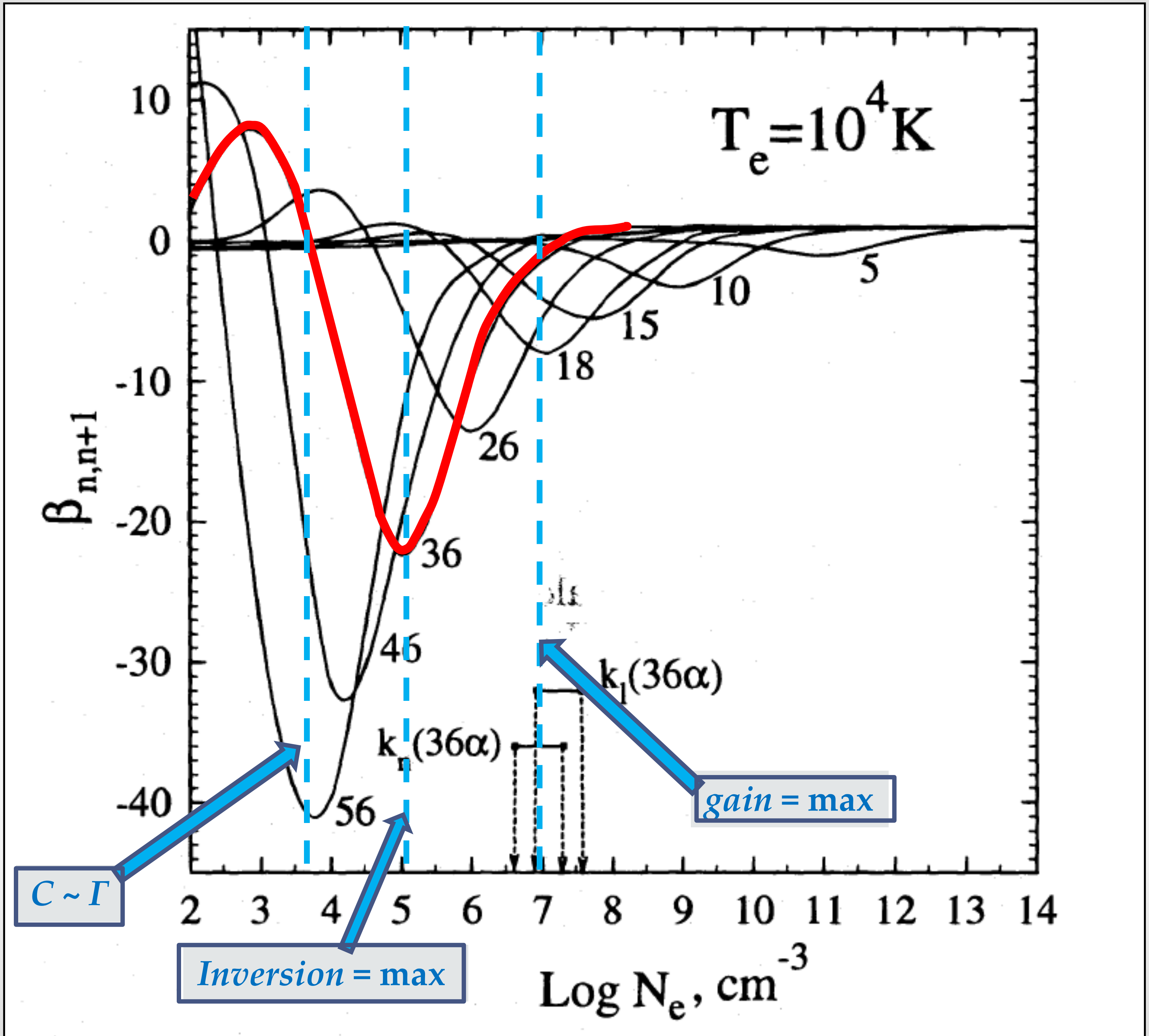
MWC 349:
The First Natural Hydrogen Maser



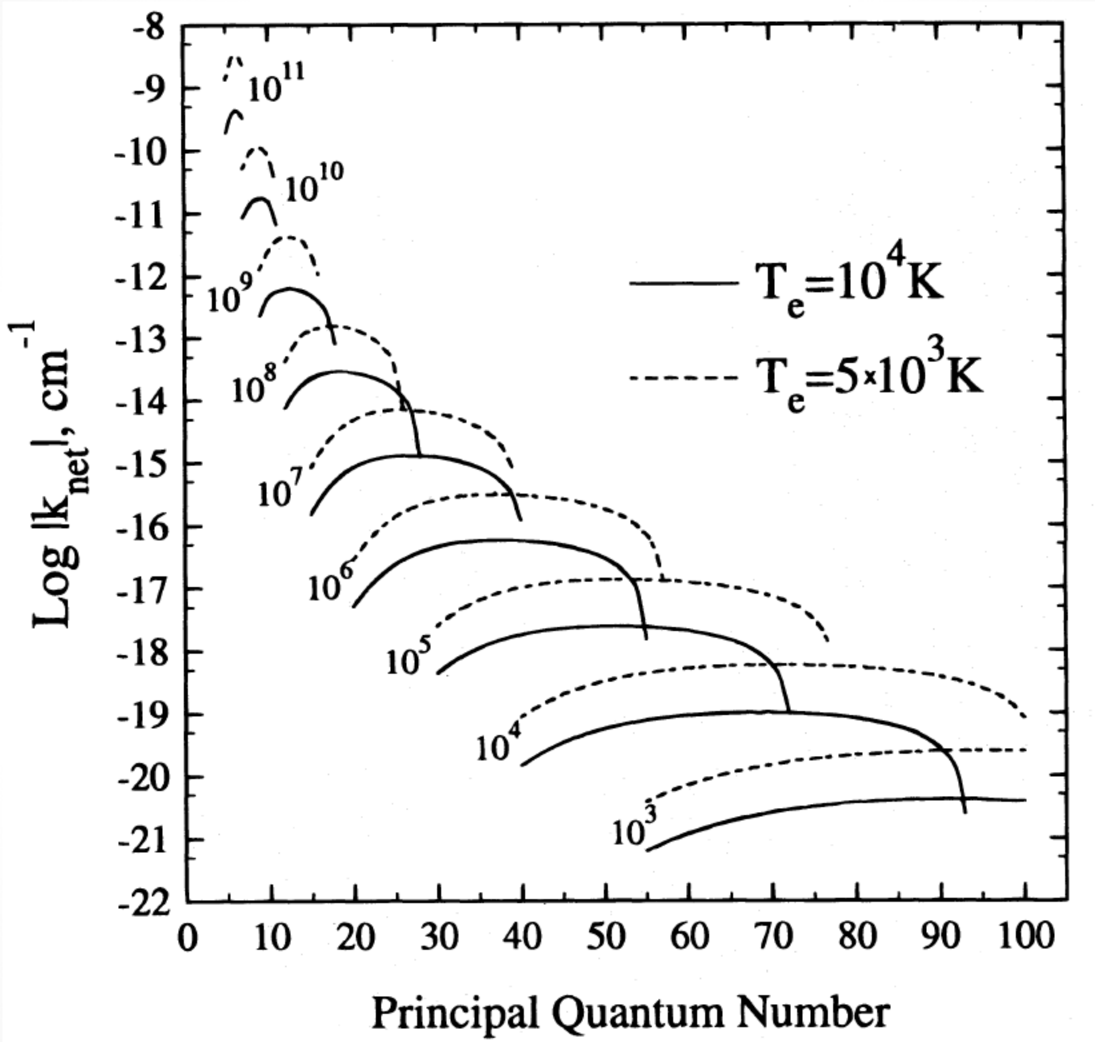
Martín-Pintado et al. (1989)



Strelnitski *et al.* (1996)
[based on computations by Hummer & Story (1992)]



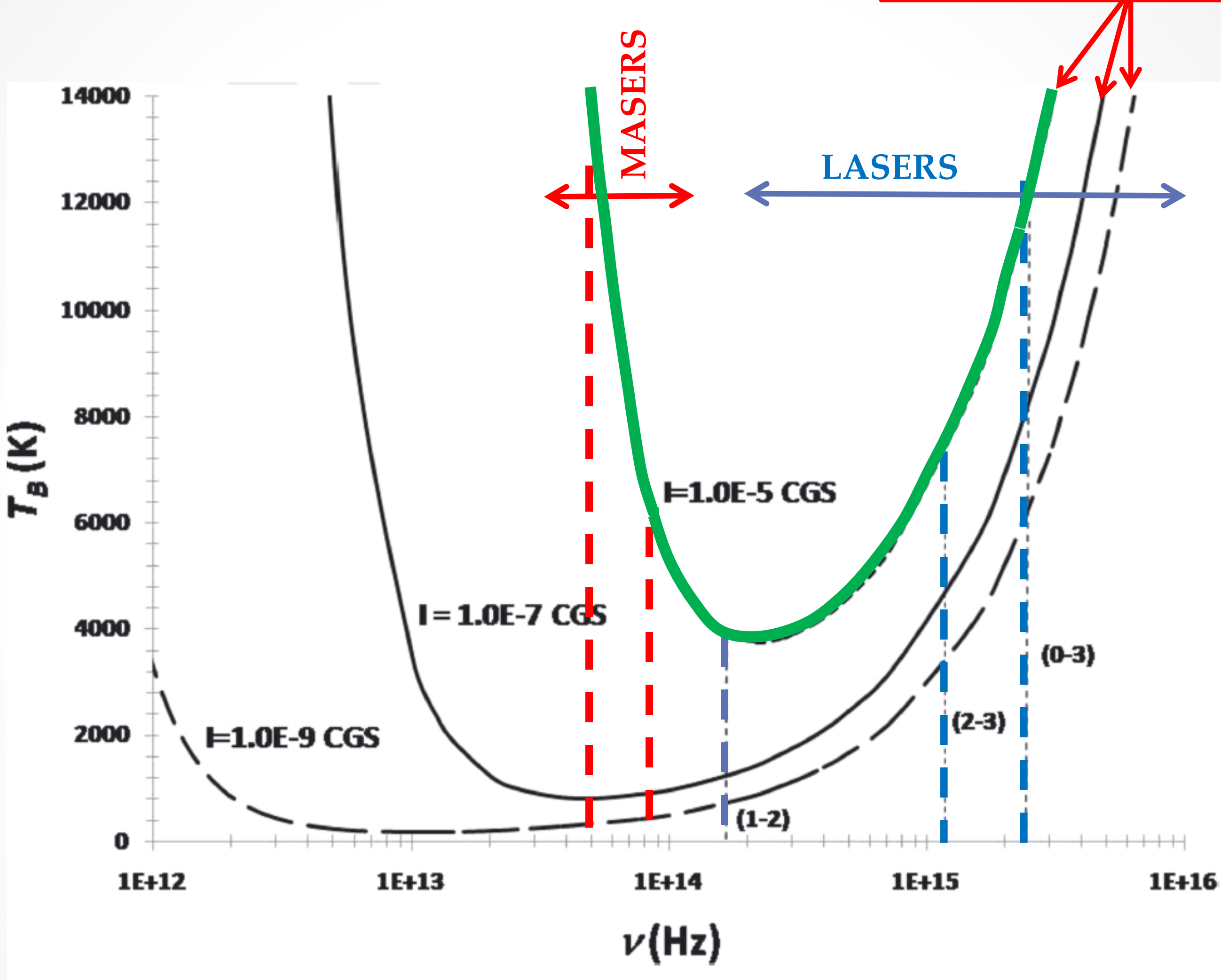
T_e=10^4 K
β_{n,n+1}
Log N_e, cm^-3
10
0
-10
-20
-30
-40
2
3
4
5
6
7
8
9
10
11
12
13
14
5
10
15
18
26
36
46
56
k_l(36α)
k_n(36α)
C ~ Γ
Inversion = max
gain = max



Log $|k_{net}|$, cm^{-1}
Principal Quantum Number
$T_e=10^4$K
$T_e=5\times10^3$K
10^{11}
10^{10}
10^9
10^8
10^7
10^6
10^5
10^4
10^3

$$I \propto N_{ph} \cdot \ h\nu \cdot \Delta\nu^{-1} \ \rightarrow I_{mas} \ \leq I_{pump}$$

Lines of Constant Intensity



Messenger & Strelnitski (2011)

Polarization

Goldreich, Keeley & Kwan (1973; GKK) :

1-D maser; magnetic field B; J=1-0 transition; isotropic pumping

4 key parameters with dimension frequency:

$\Delta\omega$ - bandwidth of radiation;

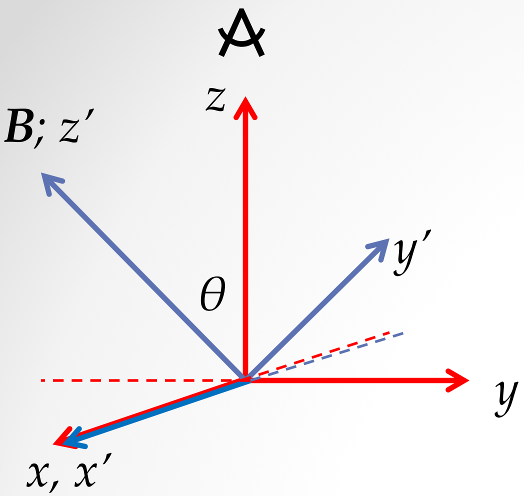
$g\Omega$ - Zeeman splitting

g - Landé g value for the upper state

$\Omega = eB_0/mc$ – girofrequency

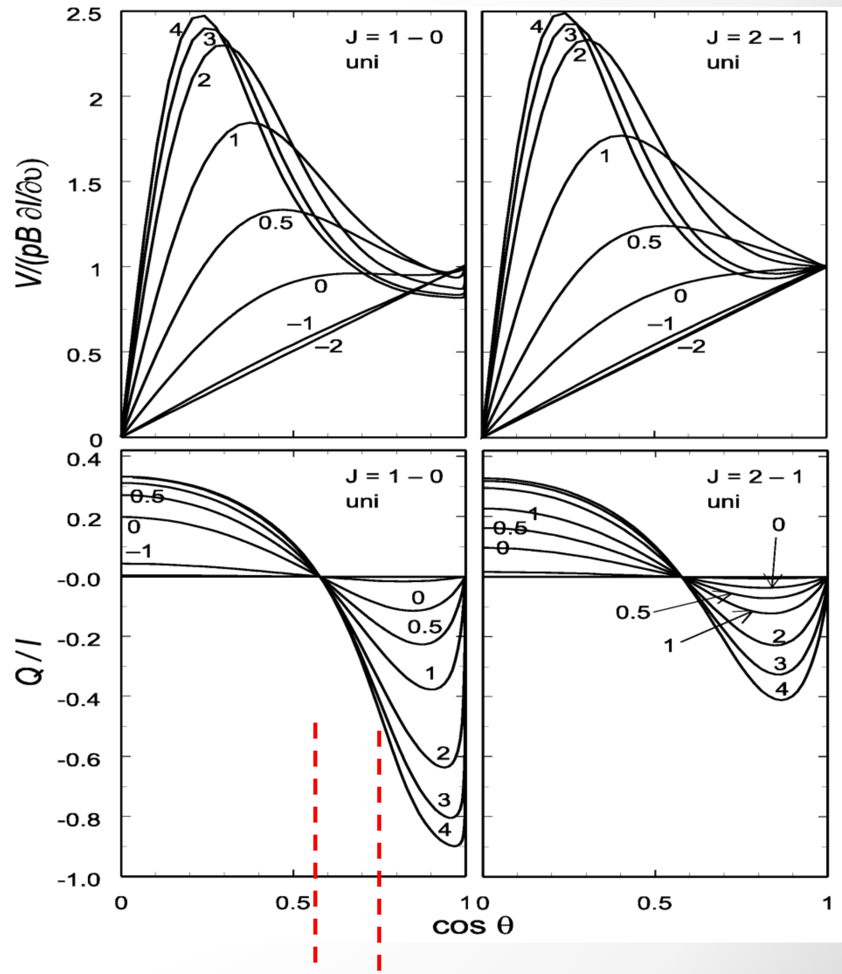
Γ - population decay rate

R - stimulated emission rate



$Q = I_x - I_y$

Watson & Wyld (2001)



55° 35°

GKK:

nonparamagnetic molecules

saturation

$$R \ll g\Omega \ll \Delta\omega, \quad R/\Gamma \gg 1$$

$$\frac{Q}{I} = \frac{3\sin^2\theta - 2}{3\sin^2\theta}$$ for $\sin^2\theta \geq 1/3$ ($\theta \geq 35°$)

$$\frac{Q}{I} = -1$$ for $\sin^2\theta \leq 1/3$ ($\theta \leq 35°$)

$\sin^2\theta = 2/3 \;\rightarrow\; \frac{Q}{I} = 0$

($\theta = 54.7°$, van Vleck angle)

Circular Polarization of H_2O Masers (Fiebig & Güsten, 1989)

$$V \propto B \frac{\partial I}{\partial \nu} \cos\theta$$

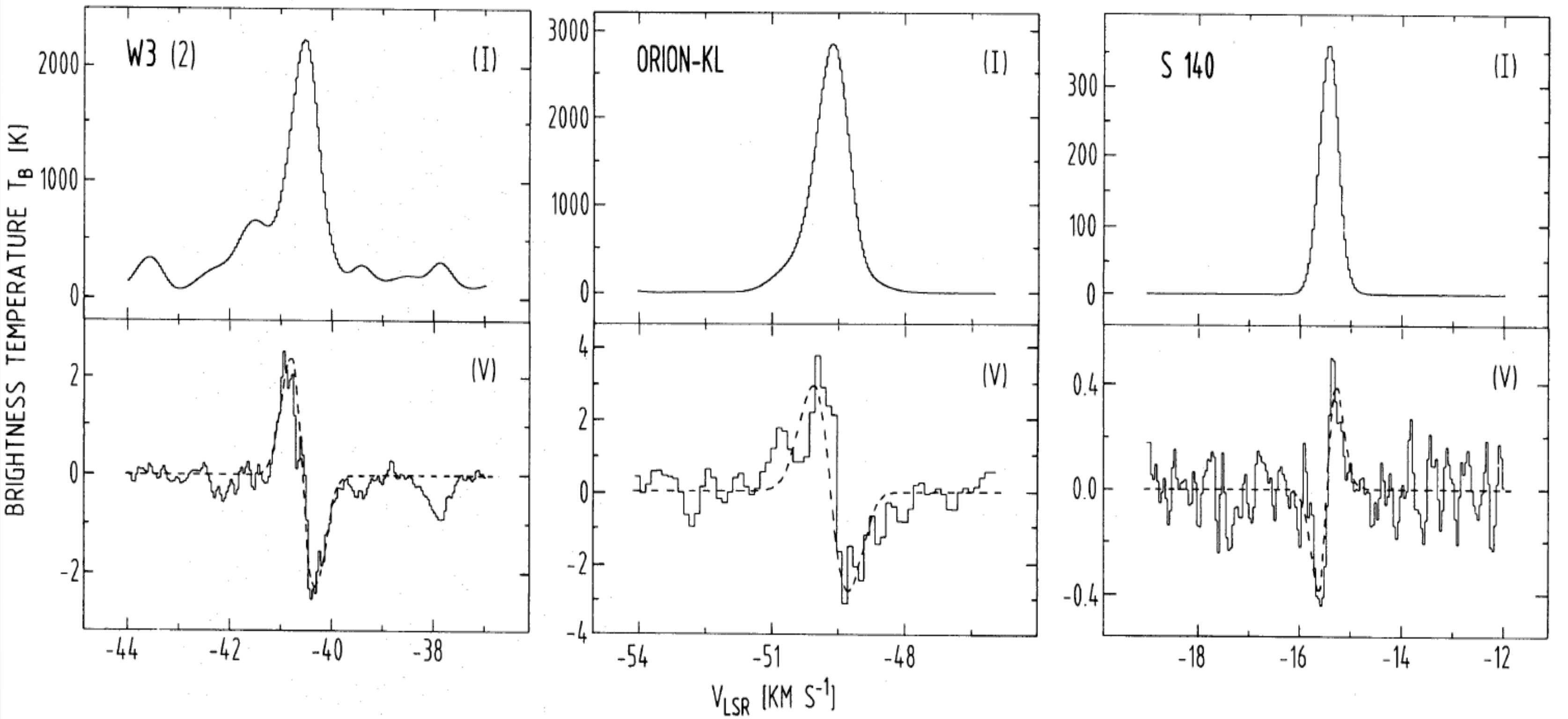


Fig. 3. Total power (I) and V-spectra ($T_{RHC}-T_{LHC}$) for selected sources (Table 1). Fits of synthetic V-spectra to the main (Gaussian) velocity component are superimposed (see Sect. 2). Note that due to differential beam coupling and hence small gain differences, scaled-down replicas of nearby total power lines show-up in the W3(2) V-spectrum

- **Non-magnetic Explanation of Linear Polarization**

  Competition between intersecting rays of a saturated maser in non-spherical media and/or media with velocity gradients + lack of axial symmetry along the line of sight causes linear polarization. (Western &Watson, 1983)

- **Non-magnetic Explanation of Linear Polarization**

  Anisotropic pumping of an unsaturated maser (e.g. Ramos & Degl'Innocenti, 2005)

- **Non-Zeeman Explanation of Circular Polarization**

  Change of the quantization axis from the direction of the MF to the direction of propagation with the increasing R, when $R \sim g\Omega$ ('Intensity-dependent polarization') (Nedoluha & Watson, 1994)

- **Non-Zeeman Explanation of Circular Polarization**

  (Observed) linear polarization + variations of the orientation of magnetic field along the line of sight (Wiebe & Watson, 1998)

When *B* is "unusually" high, think of the above possibilities!

Reviews on astrophysical maser polarization:

Watson W.D. 2009, Rev.MexAA (Serie de Conferencias), 36, 113

Elitzur, M. 2002, 2007: Reviews at the Brazil and Australia Maser Symposia

Some Prospects

- Methods of Extraction of the principal pumping cycles
- More work on “spooks” versus “things” dilemma
- Probing turbulence in H2O fountains
- More work on theory of polarization
- Theory of cyclotron masers
- Recombination lines on Sun