To: EDGES Group  
From: Alan E.E. Rogers  
Subject: Theoretical noise for 3-position switching using Acqiris DP310

Switching from antenna to a load and then to a load plus noise calibration allows the antenna temperature to be estimated from

\[ T_A = \left[ \frac{p_0 - p_1}{p_2 - p_1} \right] T_{\text{cal}} - T_{\text{load}} \]  

where 

\[ p_0 = (T_A + T_R)g \]  
\[ p_1 = (T_L + T_R)g \]  
\[ p_2 = (T_L + T_{\text{cal}} + T_R)g \]

where \( T_A \) = antenna temperature  
\( T_L \) = load temperature  
\( T_R \) = receiver noise temperature  
g = receiver gain

If the system is linear the relation of equation (1) is independent of the bandpass and any constant additive noise which is present in the total power of each switch position. The noise in \( T_A \) can be estimated from the noise in the spectra from each switch position. Since the noise in each position is independent of the noise in another position the total noise is the square root of the squares of the individual noise components:

\[ \Delta T_A \approx \sigma \left[ \left( \frac{(T_A + T_R)^2}{T_{\text{cal}}^2} \right) + \left( \frac{(T_L + T_R)^2}{T_{\text{cal}}^2} \right) + \left( \frac{(T_A - T_L)^2}{T_{\text{cal}}^2} \right) \right]^{\frac{1}{2}} \]  

where \( \sigma_0, \sigma_1 \text{ and } \sigma_2 \) are the fractional noise in each switch position. If equal times are spent in each position and \( T_{\text{cal}} \gg T_L \)

\[ \Delta T_A \approx \sigma \left[ \left( \frac{(T_A + T_R)^2}{T_{\text{cal}}^2} \right) + \left( \frac{(T_L + T_R)^2}{T_{\text{cal}}^2} \right) + \left( \frac{(T_A - T_L)^2}{T_{\text{cal}}^2} \right) \right]^{\frac{1}{2}} \]  

where \( \sigma = (bT/3)^{\frac{1}{2}} \)  
b = resolution bandwidth (Hz)  
\( T \) = total integration (s)
Parameters used by dp310

\begin{align*}
T_{\text{cal}} & \quad 400 \text{ K} \\
T_L & \quad 300 \text{ K} \\
T_R & \quad 50 \text{ K} \\
\text{Sample rate} & \quad 420 \text{ Ms/s} \\
\text{Number samples / FFT} & \quad 32,768 \\
\text{Number spectral points} & \quad 16,384 \\
\text{Number FFT blocks per integration period} & \quad 5120 \\
\text{Integration time} & \quad 0.4 \text{ s per switch position} \\
\text{Real time} & \quad \sim 10 \text{ s per switch position} \\
\text{Window function} & \quad \text{Blackman – Harris} \\
\text{Frequency spacing} & \quad 12.817 \text{ kHz} \\
\text{Frequency resolution (Blackman – Harris)} & \quad 26 \text{ kHz} \\
\text{Effective integration Fraction for Blackman – Harris} & \quad 0.5 \\
\text{Calculated rms noise for } T_A = 10^3 & \quad 24 \text{ K} \\
& \quad 26 \text{ kHz resolution and 30 seconds real time} \\
\end{align*}

(For } T_A = 500 \text{ K the rms noise is } 11 \text{ K})

\text{Calculated rms for 24 hours realtime and smoothing to 1 MHz} \quad 72 \text{ mK}

In summary the EDGES system has increased noise as a result of the following factors:

1] 96\% of the time the data is being transferred and processed so noise is increased by a factor of 5

2] Each FFT is windowed to obtain very high rejection of the resolution sidelobes this increases the noise by } \sqrt{2}

3] For the 400 K calibration noise is increased by a factor of about 1.3, depending on the antenna temperature, compared with a much stronger calibration.