

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
HAYSTACK OBSERVATORY
WESTFORD, MASSACHUSETTS 01886
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Telephone: 781-981-5400
Fax: 781-981-0590

To: EDGES Group

From: Alan E.E. Rogers

Subject: Noise wave analysis using reflection coefficients referenced to 50 ohms

The noise wave representation of the energy transfer from the antenna to the LNA has some advantages over a pure circuit model. For example consider transfer of sky noise from an antenna with impedance, Z_{ant} , to an LNA with impedance Z_{lna} . The reflection coefficient, Γ , is

$$\Gamma = (Z_{ant} - Z_{lna}^*) / (Z_{ant} + Z_{lna}) \quad (1)$$

And the noise power from sky temperature, T_{sky} , into the LNA is

$$T_{sky} (1 - |\Gamma|^2) \quad (2)$$

However it is more convenient to measure the reflection coefficients in a 50 ohm system in which case the power to the noise power to the LNA is

$$T_{sky} (1 - |\Gamma_a|^2) |F|^2 \quad (3)$$

Where

$$F = (1 - |\Gamma_\ell|^2)^{1/2} \left[\sum_{K=0}^{\infty} (\Gamma_a \Gamma_\ell L)^K \right] = (1 - |\Gamma_\ell|^2)^{1/2} / (1 - (\Gamma_a \Gamma_\ell L)) \quad (4)$$

and where Γ_a and Γ_ℓ are the reflection coefficients of the antenna and LNA respectively with respect to 50 ohms. The complex factor F is the sum of noise voltage waves back and forth between the antenna and LNA via a 50Ω attenuator or cable with one way power loss factor L. The polylogarithmic series converges when $|\Gamma_a \Gamma_\ell L| < 1$ and has an exact solution.

We can add the contributions of the noise from the attenuator and LNA to equation (3) as follows:

$$T_{sky} (1 - |\Gamma_a|^2) L |F|^2 + T_{amb} \left[(1 - L) L |\Gamma_a|^2 + (1 - L) \right] |F|^2 + T_{LNAU} |\Gamma_a|^2 L^2 |F|^2 \quad (5)$$

$$+ T_{LNAC} |\Gamma_a| L |F| \cos \phi + T_{LNA}$$

Where T_{amb} is the temperature of the attenuator. The noise from the LNA is in 3-parts are as follows:

T_{LNAU} = noise towards antenna which is uncorrelated with other sources of noise

T_{LNAC} = noise towards antenna which is correlated with other noise from LNA

T_{LNA} = noise contribution from LNA when connected to a 50 Ω load.

The correlated noise from the LNA depends on the difference between the phase of the reflection, ϕ_r , and the correlation phase, ϕ_c , where

$$\phi_r = \text{phase of } \Gamma_a F \quad (6)$$

Solving for the 4 LNA noise parameters, T_{LNAU} , T_{LNAC} , T_{LNA} and ϕ_c

- 1] Connect matched noise source to convert power to temperature
- 2] Connect matched load

$$T = T_{amb} + T_{LNA} \quad (7)$$

- 3] Connect open cable with measured Loss L so $\Gamma_a = e^{-2i\omega\tau}$ where ω is the angular frequency and τ the cable's delay.

$$T = T_{amb} \left[1 - L^2 \right] |F|^2 + T_{LNAU} L^2 |F|^2 + T_{LNAC} L |F| \cos(\text{phase of } \Gamma_a F - \phi_c) + T_{LNA} \quad (8)$$

$T_{amb} \left[1 - L^2 \right] |F|^2$, is known from the measurements the cable and the LNA input impedances, along with T_{LNA} can be subtracted from equation (8). The remaining terms can be found by fitting functions of $L^2 |F|^2$, $L |F| \cos(\text{phase of } \Gamma_a F)$ and $L |F| \sin(\text{phase of } \Gamma_a F)$ to determine T_{LNAU} , T_{LNAC} and ϕ_c . For EDGES the following are typical results at 100 MHz:

$$T_{LNA} = 43 \text{ K}$$

$$T_{LNAU} = 40 \text{ K}$$

$$T_{LNAC} = 37 \text{ K}$$

$$\phi_c = 108 \text{ degrees}$$

In summary measurements made of antenna and LNA input impedance along with EDGES measurements of an open cable to derive the LNA noise waves can provide a complete calibration of EDGES.