To: EDGES Group  
From: Alan E.E. Rogers  
Subject: Set of basis functions needed to account for EDGES systematics

The dominant source of systematic errors expected in the EDGES system with absolute calibration are:

1. Errors in S11 measurements (see memo #91)  
2. Errors in measurement of temperature (see memo #91)  
3. Errors due to the frequency dependence of the beamshape (see memo #71)  
4. Errors due to the ionosphere (see memo #79)

These errors limited the ability to measure the true sky noise spectrum in order to detect or set limits on a signature from the 21-cm line in the frequency range 50 to 200 MHz. Since the errors cannot be perfectly modeled functional parameters are needed in the weighted least squares analysis in order to accommodate these errors. In the past polynomials were used to account for the errors. However it is an advantage to use as few functions as possible and these should be chosen to best model the errors. A set of only 6 basis functions has been found to adequately model the errors and maximize the signal to noise ratio of the measurement of a 21-cm line signature assumed to be a Gaussian with a strength of 30 mK and full width at half power of 50 MHz centered at 145 MHz. The key functions are:

1. A scale factor correction to the sky temperature spectral index  
   \[ f_1(f) = f^{-2.5} \]

2. A constant  
   \[ f_2(f) = 1 \]

3. A function to account for the frequency dependence of the beam  
   \[ f_3(f) = f^{-0.5} \]

4. Two functions to account for the contribution for the ionosphere to the sky temperature  
   \[ f_4(f) = f^{-4.5} \]
   \[ f_5(f) = f^{-2.0} \]

5. A function to model the 21-cm line  
   \[ f_6(f) = e^{-0.69(f-145)^2/25^2} \]

Simulations show that the 6 parameter fit can extract the 21-cm line of 30 mK with SNR of 9 in 10 days observing of 6 hours of nighttime observing per day in a frequency band.
from 100 to 190 MHz. Without the need to parameterize and solve for the systematic errors terms the 21-cm line of 30 mK could be detected in with SNR of 9 in one 6 hour session. The detectability of a 100 mK line at 72.5 MHz with half-power full width of 16 MHz is about equally detectable. The factor of four increase in the ionosphere and the factor of six increase in sky noise are offset by the threefold increase in line strength and a 50% reduction in fractional line width.

The simulations were done using the beam patterns of the antenna with Roberts balun (described in memo #89) using FEKO. While no corrections were made for the frequency dependence of the dipole of orientation az = 60° perpendicular to line between elements was found minimize the frequency dependence when convolved with the Haslam sky map for the latitude of Boolardy. Also the data was averaged over 0 to 12 h LST. In practice it would be better to correct the data for each LST. For this simulation a constant spectral index of -2.5 was assumed. A preliminary test shows that the effect of the spatial dependence of the spectral index on the ability to extract the 21-cm signal is small but further study is needed.

It is noted that introducing a function of $f^{0.5}$ to account for frequency dependent ground and antenna loss increases the covariance for the determination of the 21-cm line by a factor of 100 so that it would take 1000 days to reach an SNR of 9. Consequently adding this function is unacceptable and we need to model the ground loss to within about 3 K or keep the loss below this level which requires a large ground plane (see memo #88). Another systematic similar to ground loss is that of an elevated horizon which then results in a frequency dependent contribution. A 5° horizon contributes about 300 mK and has a negligible effect but a 10° horizon contributes 2.5 K and needs to be modeled and removed to an accuracy of about 25% to have little effect on the 21-cm line. While a 25° horizon would be difficult to model with sufficient accuracy it may be possible to model the smaller effect of a mountain which presents a horizon rising to 25° over a limited range of azimuth as would be the case if EDGES were deployed to the north west coast of the remote island of Tristan da Cunha in order to reach a sufficiently low level of RFI.