

**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**  
**HAYSTACK OBSERVATORY**  
**WESTFORD, MASSACHUSETTS 01886**  
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*Telephone: 781-981-5400*  
*Fax: 781-981-0590*

To: EDGES Group

From: Alan E.E. Rogers

Subject: Bench tests of cable loss between antenna and LNA

1] EDGES reference plane

Like a VNA, EDGES calibration is made at a reference plane. At this plane measurements are made of the S11 of the antenna, hot load and cold load, as well as the open and shorted cable used for noise wave calibration. Looking in the other direction from the reference plane a measurement is made of the S11 looking into the LNA. Calibration spectra are taken of the hot and cold loads as well as the open and shorted cable. To within a scale factor which is determined via calibration the 3-position switched power is given by

$$\left[ (TL + (1-L)T_{amb})(1 - |\Gamma_a|^2) + T_u |\Gamma_a|^2 \right] |F|^2 + (T_c \cos \phi + T_s \sin \phi) |\Gamma_a| |F|$$

$$\text{Where } F = \left(1 - |\Gamma_a|^2\right)^{1/2} / (1 - \Gamma_a \Gamma_\ell)$$

$\Gamma_a$  = reflection coefficient from reference plane looking away from LNA

$\Gamma_\ell$  = reflection coefficient from reference plane looking into LNA

$$\phi = \text{atan}(\Gamma_a F)$$

L = loss factor

$T_{amb}$  = ambient temperature

T = antenna or calibration temperature

$T_u$  = uncorrelated noise wave

$T_c, T_s$  = cosine and sine components of noise wave

The expression above assumes that the bandpass and 2<sup>nd</sup> stage noise have already been removed by 3 position switching. It can be shown that like a VNA the final calibrated measurements of T is independent of the actual location of the reference plane as long as all S11 measurements are made from the same reference plane. The values of  $T_u, T_c, T_s, \Gamma_a,$  and  $\Gamma_\ell,$  which along with the scale factor are determined from the calibration, will change if the reference plane is moved.

2] Loss factor

A loss factor of 1 is assumed for ambient load and noise wave calibration as its value has no effect since T equals  $T_{amb}$ . An accurate loss factor is needed for the hot load and the antenna.

This is straightforward for the hot load since the load is well matched and the loss is small. The loss factor for the antenna needs to be estimated based on the theory given in memo 125.

$$L = \text{Re}(V_{out} I_{out}^*) / \text{Re}(V_{in} I_{in}^*)$$

Where  $V_{in} = (e^{\gamma \ell} + \Gamma e^{-\gamma \ell})$

$$I_{in} = (e^{\gamma \ell} - \Gamma e^{-\gamma \ell}) / Z_c$$

$$V_{out} = 1 + \Gamma$$

$$I_{out} = (1 - \Gamma) / Z_c$$

$$\Gamma = [(Z_s - Z_c) / (Z_s + Z_c)] e^{2\gamma \ell}$$

$Z_s$  = impedance of antenna measured from the reference plane.

$Z_c$  = complex impedance of cable to antenna

$\gamma$  = complex propagation constant

$\ell$  = physical length of cable

As discussed in this memo this is not straight forward since transmission lines have a complex impedance with significant imaginary component at the low frequency range of EDGES.

### 3] Simulations of the effect of loss factor errors

Software simulations were used to verify that the reference plane can be moved and that an arbitrary 2-port placed between the reference plane and the 3-position switch has no effect on calibrated data. In this respect the system behaves like a VNA. Test of simulated data with sky temperature of 300 K at 150 MHz and -2.5 spectral index data with perturbed values indicated that a 10% error in the balun model produces residuals of 60 mK after removal of a scale error which is comparable to residuals produced by VNA errors of 0.01 dB. A full simulation using the measured S11 data for antenna, and LNA followed by a solution for a 20 MHz wide EoR signature at 150 MHz was made.

Table 1 shows the 5 most significant of sources instrumental bias using the 9 basis function listed in memo 118.

EoR bias (mK)	Contributor	Magnitude
27	VNA ant. S11	0.01 dB + 0.01 dB/100 MHz
12	VNA ant. S11	0.01 dB
11	VNA ant. S11	0.1 deg.
15	Temperature drift	1 K
19	Balun loss estimate	10%

Table 1. Instrumental bias values for EoR signature determination using only the basis functions needed to remove the foreground and ionosphere.

It is clear from Table 1 that additional basis functions are needed to remove the instrumental biases. Adding a function which equals the effect of a change of 0.01 dB in antenna S11 magnitude and another for 0.1° of antenna S11 phase. Is sufficient to reduce the bias in EoR to under 10 mK for all anticipated instrumental errors. Further reduction to under 4 mK can be accomplished by adding another 2 basis function which characterize the antenna  $|S_{11}|^2$  and the balun loss can be added with very little increase covariance for the determination of an EoR signature with a 20 MHz width.

#### 4] Antenna simulator with exaggerated loss

The frequency structure in cable loss is most pronounced (in dB) for a small diameter cable of about one wavelength long. An initial bench test antenna simulator was made by adding a box with a coiled up 1.44 meter long RG-316/U cable. This box was placed in series with the hot filament source used for the tests described in memos 104 and 120.

Figure 1 is a photograph of this box with its cover removed. The ferrite choke is used to suppress a common mode resonance. Another ferrite core on the LNA input is used to reduce the pick-up of RFI signals in the noisy environment of the Haystack control room. This RFI is so strong that common mode signals on the coax connections can leak into the coax since the required shielding of the coax is well over the typical 90 dB of the coax.

The first test was made with the filament at ambient temperature. In this case the calibrated result should equal the ambient temperature and be independent of the cable loss factor since the temperature of the filament and cable are the same. However, the calibrated result still depends critically on the S11 of the artificial antenna measured from the reference plane. The results of this test are a calibrated spectrum of 300 K with rms of 500 mK. The rms is a combination of systematics due to error in the S11 measurement and noise. The reflection coefficient is in the range of -4 to -2.5 dB from 50 to 200 MHz for the cold filament so that an error of 0.01 dB corresponds to about 750 mK.

In the second test, with the filament heated to 1575 K, the residuals to a single parameter fit of a scale factor are given in the table below which shows the impact of the added cable. The fit using a simple cable model which the loss is proportional to the square root of frequency had a rms of 21 K after searching for best value of loss at 150 MHz. A fit using the full model of the cable with complex impedance was not much better until an adjustment was made to the conductivity to the value given in the table. This assumed a fixed cable length equal to the measured length.

Average (K)	rms (K)	(s/m)	Comments
1347	65	-	No cable correction
1579	21	-	Best fit loss correction of 0.45 dB
1580	8	$2.5 \times 10^7$	Best fit cable model
1580	4	$2.5 \times 10^7$	Best fit cable model + S11 parameters

Table 2 RMS residuals of calibrated data from hot filament

A best fit conductance well below the value of  $6 \times 10^7$  s/m is probably the result of penetration below the 40 micro inch depth of the silver plated copper (SPCW) in to the steel since the skin

depth at 100 MHz is over 250 micro inches. As a result the test needs to be repeated with a cable whose center conductor is pure copper or silver plated copper (SPC)

After an exhaustive search, a SPC cable was obtained courtesy of Micro-wax who kindly supplied a 5 foot sample of their UT-085C-TP semi-rigid cable which is shown in Figure 2. This cable was used between the hot filament source and the EDGES-2 spectrometer (serial number 1). The following results were obtained:

Average (K)	rms (K)	(s/m)	Comments
1390	54	-	No cable correction
1574	13	0	Best fit low correction of 0.36 dB
1557	7	$5.9 \times 10^7$	Best fit cable model
1558	4	$5.9 \times 10^7$	Adjusted
1558	0.6	$5.9 \times 10^7$	Model + S11 parameters

Table 3 RMS residuals of calibrated data using UT-85C cable.

In this case the cable model using the dimensions on the data sheet the corrected data fit a constant with a rms residuals of 7 K and adjusting the conductivity only increased the residuals. However it was found that a small adjustment of the cable center conductor diameter from 0.51 mm to 0.49 m reduced the residuals to 4 K and adding a constant and slope correction to the S11 brought the residuals down to 600 mK.

A closer examination of the sources of possible error in the cable model was made by first comparing the S1 of the UT-085 cable with an open and a short. The measured S11 agrees with the model with a rms residual of 0.01 dB and 0.5 degrees for a best fit to the center conductor radius of  $0.508 \pm 0.01$  mm 3-sigma. The best fit physical length of the cable plus connectors was 61.8" so that the model has to be composite of 60" of 0.085 plus 1.8" of connectors and adaptors as well as the open and short. A more complete model was tested assuming the 1.8" was equivalent to 0.141 semi-rigid. The composite S11 is calculated by conversion of the S11 of each section to impedance for the load to the next section. The composite model made very little difference, however when the S11 is modeled for the combination of the 0.085 cable and the hot filament load the composite model did improve the fit by a small amount but even after adjusting the center conductor radii best fit had an rms of 0.02. dB.

Figure 3 shows the S11 the 0.085 cable attached to the hot filament source along with the difference of S11 magnitude between the measured S11 and the composite model. Part of the difference is due to errors in the S11 measurement because repeated measurements result in changes of about 0.01 dB which is being studied. Repeated measurements show that S11

measurements of the hot filament source need to be made after the filament has been on long enough for the leads have fully stabilized to the thermal environment.

The loss estimate for antenna balun tube is about 0.02 dB at 150 MHz. The 4" flexible cable connecting the balun tube to the LNA adds about 0.015 dB. Failure to treat the combination as a "composite" changes the estimate by 0.002 dB. In summary, despite some issues of S11 accuracy, these results provide confidence in the ability to model the balun cable whose loss is an order of magnitude smaller and whose variation with frequency is 2 orders of magnitude smaller.

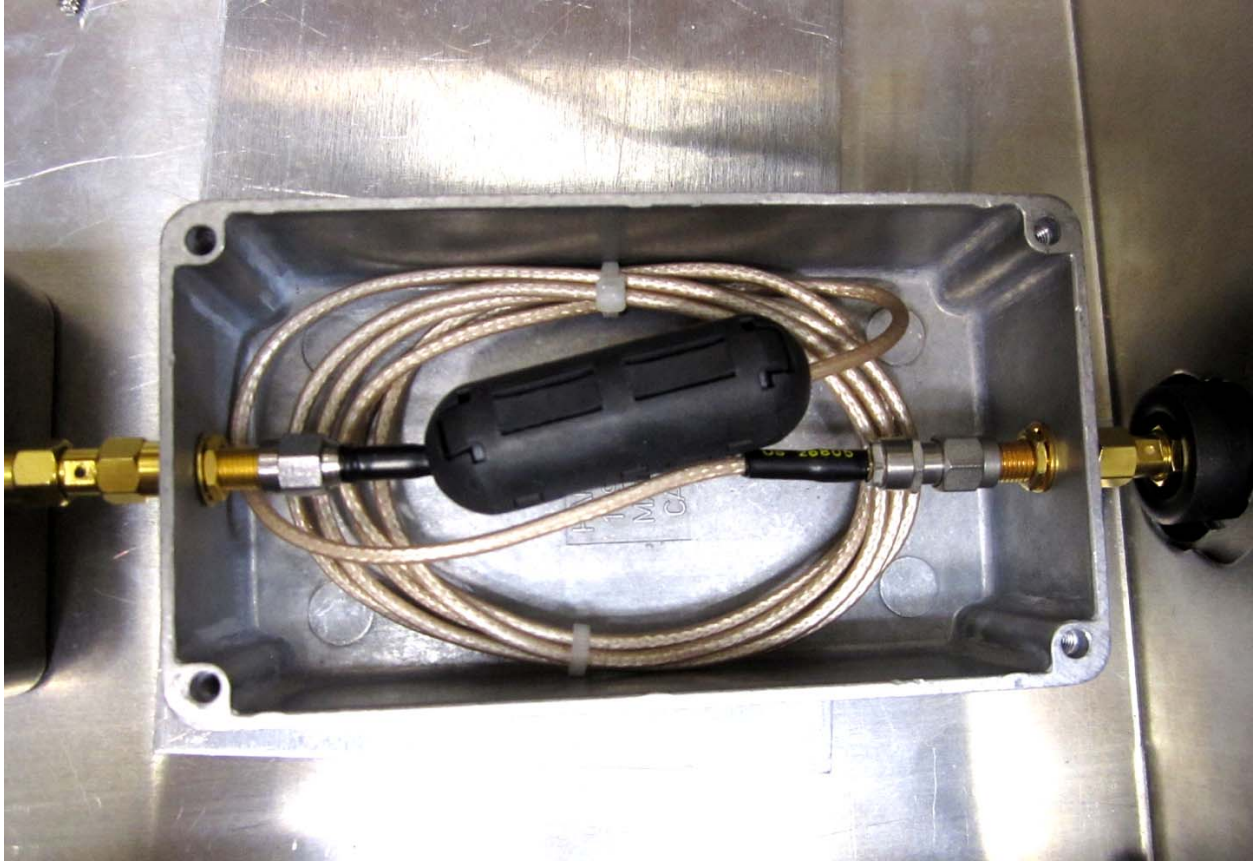
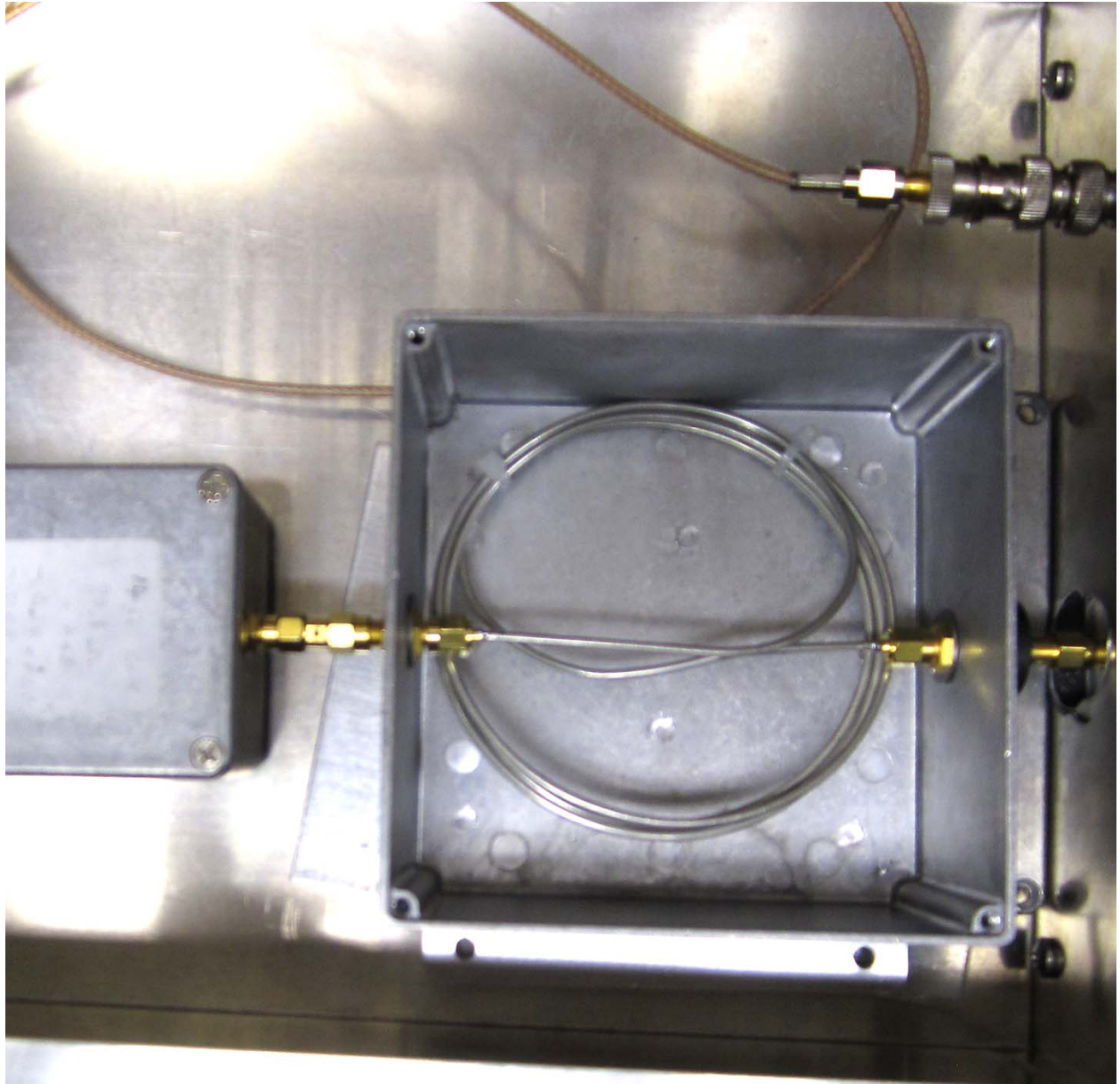


Figure 1. Photo of cable between filament source and EDGES input.



Figures 2 Semi-rigid cable between filament source and EDGES input.

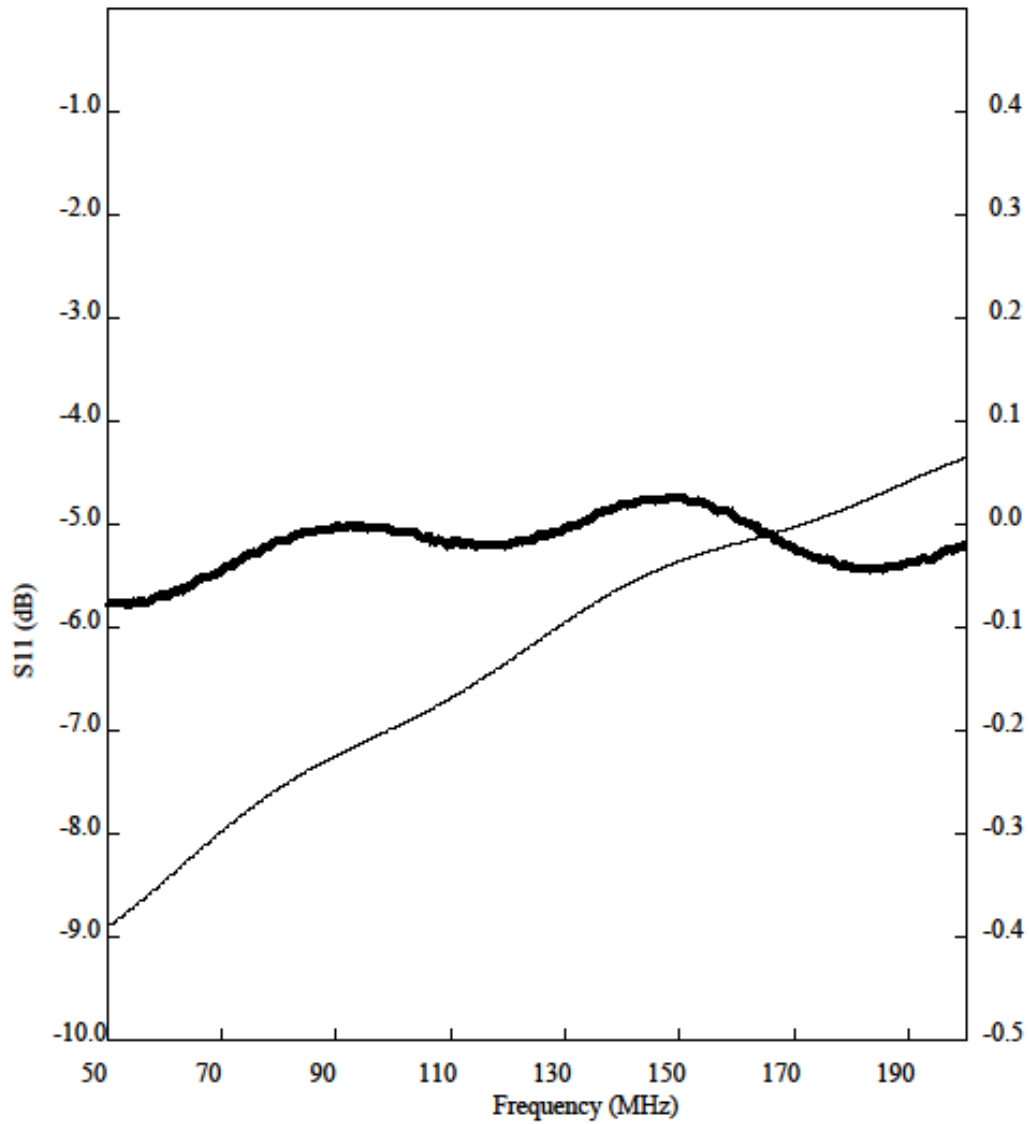


Figure 3. S11 of the hot filament source through the UT-085 cable. The thick line is the difference between the measured S11 and calculated S11 based on a model of the cable. The difference is plotted on the expanded scale on the right hand side of the plot.



Appendix:

Composite cable S11 and loss

If  $cab(f, d, \Gamma, m)$  is a complex function which returns the S11 of a cable connected to a load with  $S11=\Gamma$  then

$$cab(f, d1, cab(f, d2, \Gamma, m2), m1)$$

Returns the S11 for a composite cable of sections 1 and 2.

Where  $f$  = frequency

D1 and d2 are the physical lengths

m1 and m2 are the cable characteristics

$\Gamma$  = S11 of input

The loss for a composite cable is just the product of the losses for each section after the appropriate S11 transformation is made using the  $cab()$  function to get the S11 of the first section that constitutes the load seen by the second. Both the loss and cable functions make the transformation from S11 referred to  $50\Omega$  to S11 referred to the cable impedance internally.

i.e. The composite loss is

$$loss(f, d1, \Gamma, m1)loss(f, d2, cab(f, -d1, \Gamma, m1), m2)$$

Where m1 is the cable section closest to the LNA and  $\Gamma$  is the S11 looking out of the LNA.

Approximations in cable model:

From the full theory of Ramo and Whinnery equation 6-10 (6)

$$Z_i = -TJ_0(Tr_0)/(2\pi r_0 \sigma J'_0(Tr_0))$$

Where  $z_i$  = internal impedance of a cylindrical conductor of radius  $r_0$  and conductivity  $\sigma$

$$T = (-j\omega\sigma\mu) \text{ equ. 6-10 (2)}$$

$J_0$  = Bessel function of complex argument

$J'_0$  = derivative of  $J_0$

The problem of using the full theory is that above about 20 MHz the Bessel function series requires extreme precision which results in a loss of accuracy. This problem is pointed out by Jim Lesurf of St. Andrew's University. Lesurf gives an improved approximation to the high frequency limit for which

$$\begin{aligned} \operatorname{Re} Z_i &= R_0 \left( \left( q/2\sqrt{2} \right) + 0.26 \right) \\ \operatorname{Im} Z_i &= R_0 \left( \left( q/2\sqrt{2} \right) - 0.02 \right) / w \end{aligned}$$

Where  $q = 2^{1/2} r_0 / \delta$

$$R_0 = 2^{1/2} / (\pi r_0 q \sigma \delta)$$

$$\delta = 1 / (\pi \mu f \sigma)$$

The incorporation of Lesurf's corrections makes a small improvement in the fits of the cable model to S11 at the level of a few percent. For the 5' 0.085 cable the internal impedance values at 100 MHz are

$$2.116 + 2.0917j \text{ ohms}$$

With the corrections and

$$2.0935 + 2.0935j \text{ ohms}$$

Without corrections which corresponds to about a 1% difference.