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To: EDGES Group

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Subject: 21-cm absorption signature for large optical depth

The low band 21-cm absorption signature used for detection and detection limits is of the form

$$T(\nu) = -a\exp\left(-ln(2)(\nu-\nu_0)\right)^2 / \left(w^2/4\right)$$

where a = peak amplitude of absorption dip

ν = frequency

ν_0 = center frequency of absorption

w = full width at half maximum (FWHM)

For a significant optical depth the "dip" can become flattened. For an optical depth τ the brightness temperature is proportional to

$$\left[1-\exp\left(-\tau(\nu)\right)\right]$$

Via the theory of radiative transfer.

If the optical depth has a Gaussian frequency distribution

$$T(\nu) = -a\left[1-\exp\left(-\tau_0\exp\left(-(\nu-\nu_0)^2 b/\left(w^2/4\right)\right)\right)\right] / \left(1-\exp(-\tau_0)\right)$$

where $b = -\ln\left(-\ln\left(\left(1+\exp(-\tau_0)\right)/2\right)/\tau_0\right)$

and τ_0 = maximum optical depth

For small opacity the expression is a Gaussian with the same width as the Gaussian width of the optical depth. As τ increases the absorption flattens and the width increases. Table 1 shows the fractional increase as a function of the maximum opacity.

Opacity	Fractional width
0.1	1.02
1	1.18
2	1.35
4	1.60
6	1.77
8	1.88
10	1.96

Table 1. Fractional width increase with opacity

Figure 1 plots a simulation of 0.5 K deep absorption of 20 MHz FWHM for maximum optical depth of 1,2,4,6,8,10. Figure 2 and 3 show the same simulations with 3 and 4 physical terms removed respectively. It is noted that:

1] With 3 terms removed the residual signature changes with optical depth but is still recognizable and the rms increases a little.
2] With 4 terms removed the change in the residual signature is dramatic. At $\tau = 4$ a "dip" at 70 MHz and another at 85 MHz appear.

If the collapsing regions of primordial hydrogen forms in clumps the opacity of the clumps is proportional to inverse of the clump radius squared for clumps of the same mass. While the individual clumps may not have large optical depth the 21-cm line absorption is the cumulative effect of many clumps along the line of sight.

If each clump has a large optical depth it will have a narrow width and will be just one member of a "forest of 21-cm absorption profiles" which cover the range of redshift along the line of sight. However, the forest of 21-cm profiles will have a "flattened" signature due to the saturation effects of large optical depth. A uniform spherical "clump" with density of 10^4 hydrogen atoms cm^{-3}, 2 ly in radius with spin and kinetic temperature of 500 K (approximate numbers from "Fragmentation of first gaseous objects" in Barkana and Loeb astro-ph/0010468) has an optical depth of 8 and thermal line width of 23 kHz.

Tests on the low band data find "dips" at 70 and 84 MHz when 4 terms are removed. These data can also be fit with a single absorption signature about 20 MHz wide that is flatter than a Gaussian like those which might be formed due to the saturation effects of high optical depth.

Tests for the presence of "flattened" signature have been made of the low band data from 2015_286 to 2016_035 for which the reference case was

Frequency range	60-99 MHz
GHA range	4-16 hours
Antenna S11	2015_342
Beam correction	blade 11_3.5_1e-2

and for data from 2016_259 to 2016_327 for which the reference case was

Frequency range	60-99 MHz
GHA range	4-16 hours
Antenna S11	2016_175
Beam correction	blade 9-perf7

data	case	#terms	SNR	center MHz	amplitude K	width MHz
A	reference	4 poly	38	78	0.6	19
B	reference	4 poly	24	78	0.6	19
A	reference	4 phy	16	78	0.4	19
B	reference	4 phy	14	78	0.5	20
A	no beam correction	4 poly	22	77	0.6	17
B	no beam correction	4 poly	15	78	0.6	20
A	no loss correction	4 poly	11	77	0.5	19
B	no loss correction	4 poly	27	77	0.7	18

Table 1. Results of search for signature in nighttime low band data. A and B corresponds to the data with the original and extended ground plane respectively.

A value of tau =7 was used for all cases in Table 1. Memo 217 shows possible detections of signature with widths of about 20 MHz obtained from high band with 3 parameters removed. It emphasizes the sensitivity to the sensitivity to systematics when only 3 parameters are removed and discussed the potential of using the "Galaxy up" data to reduce the sensitivity to instrumental errors. Using the same data, GHA range and calibration as in Figure 12 of memo 217 the following signature

# terms	SNR	center MHz	amplitude K	width MHz
3 phy	16	80	0.5	18
4 poly	5	79	0.2	17
3 poly*	17	78	1.53	28

Table 2. Results of a search for a signature with tau=7 in the difference spectrum of Figure 12 of memo 217. Last entry is for a Gaussian signature.

In this case the data is too noisy, owing to the very limited amount of nighttime Galaxy up data available, for a significant SNR with 4 terms removed. The last entry in Table 2 is the result of a search for a signature without any "flattening" due to large optical depth. The fit is equally good but leads to a large amplitude. With the current data it is not possible to distinguish between a flattened signature of about 0.5 K in amplitude or a Gaussian signature with a much larger amplitude.

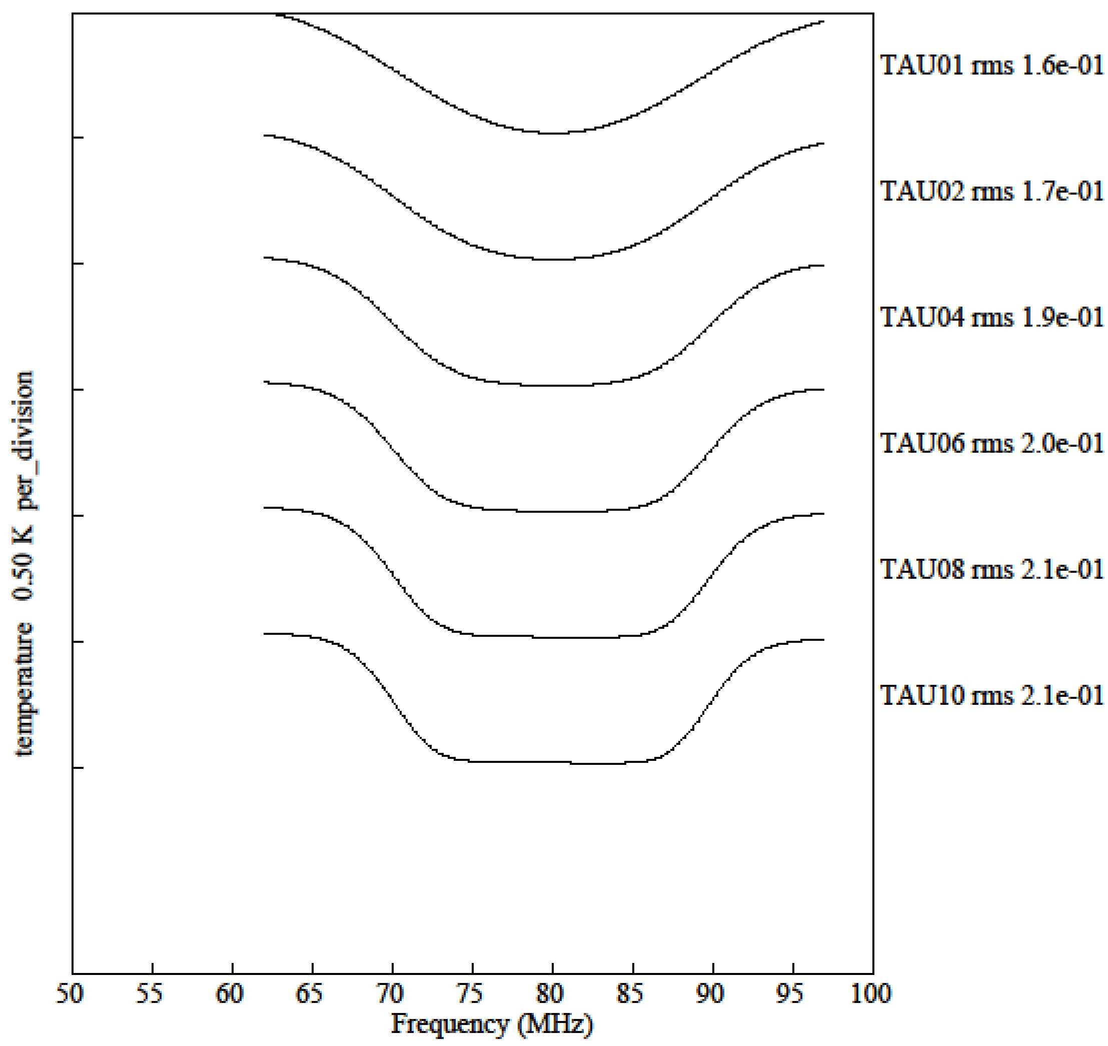

Figure 1. Signature of absorption for optical depths 1, 2, 4, 6, 8 and 10. FWHM of 20 MHz centered at 80 MHz with 0.5 K depth.

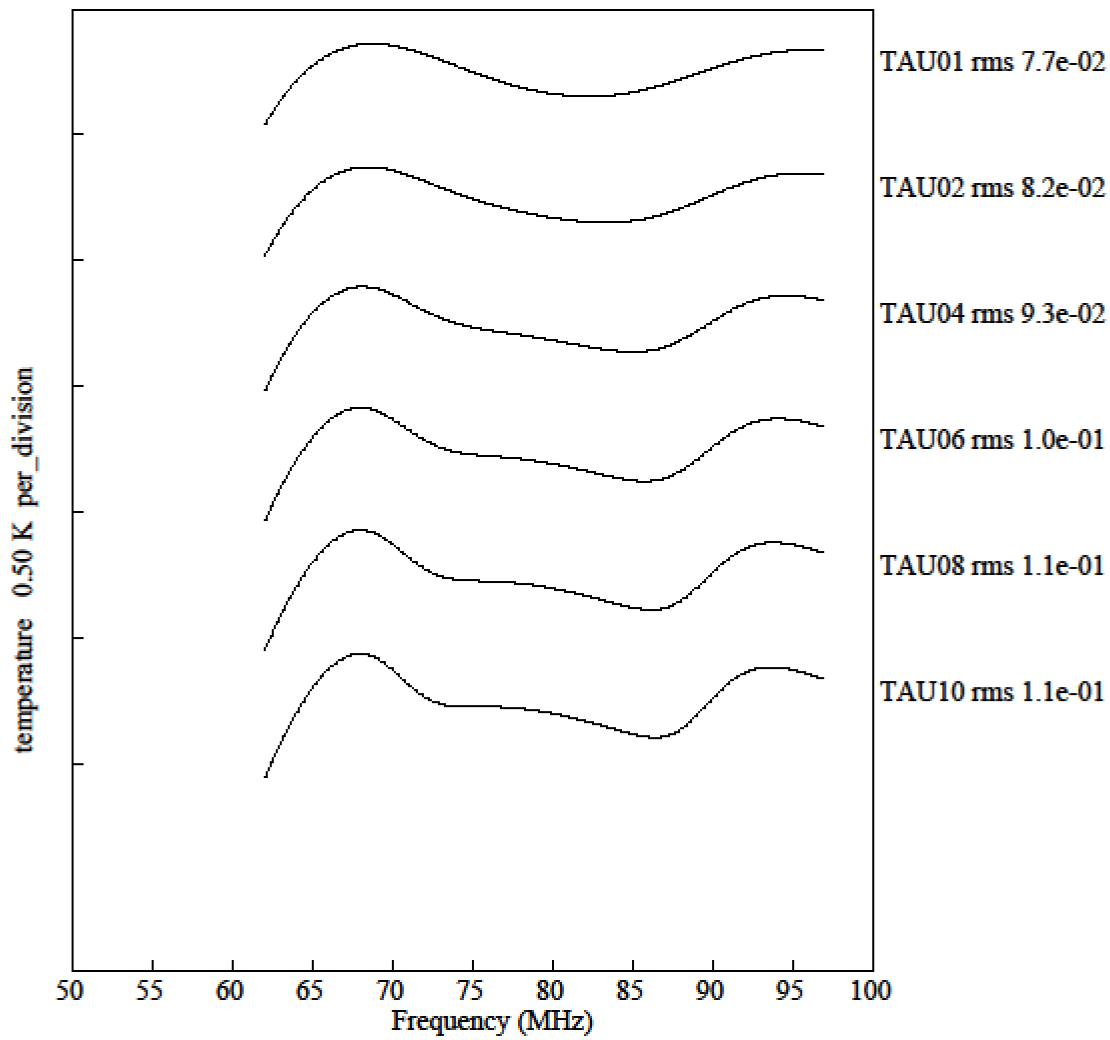

Figure 2. Residuals of signatures after removing 3 physical terms.

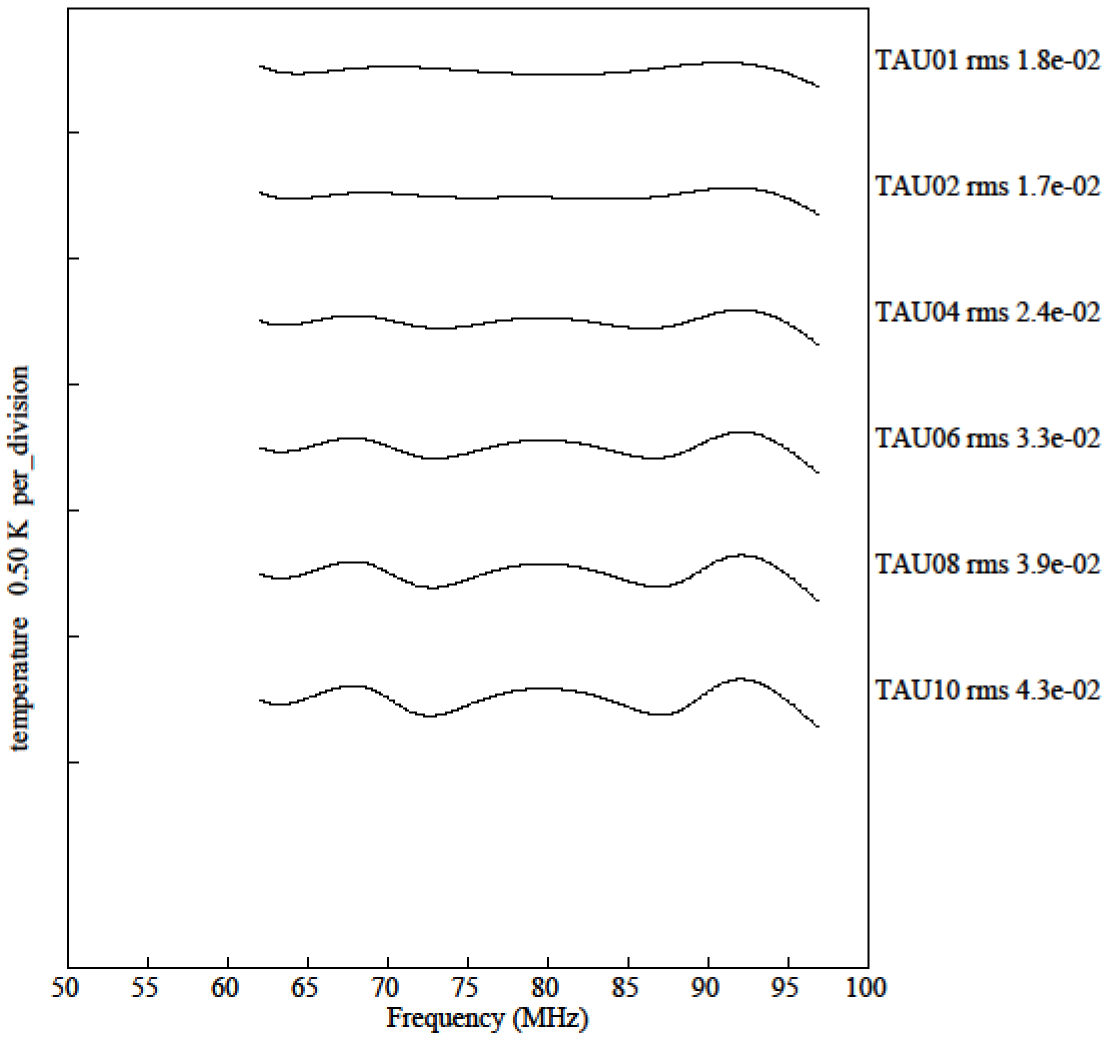

Figure 3. Residuals of signatures after removing 4 physical terms.