To: EDGES Group  
From: Alan E.E. Rogers  
Subject: Estimate of the chromaticity of the foreground reflected from the lunar surface.

The lunar surface has been studied by radar and by reflections from the lunar surface using the 136 MHz telemetry signals from Explorer 35 (Tyler G.L. Oblique-scattering radar reflectivity of the lunar surface: Preliminary results from Explorer 35 (Tyler G.L. Oblique-scattering radar reflectivity of the lunar surface: Preliminary results from Explorer 35, JGR 73 no 24 (1968) 7609-760).

The explorer 35 results are consistent with a regolith dielectric of 3.0 and conductivity of 2×10⁻⁴ S/m and an underlying rock with dielectric 8.5 and conductivity of 2×10⁻² S/m. The effects of the moon have been estimated as follows:

1] FEKO is used to model the beam of a short dipole consisting of a 0.5 m diameter sphere separated by 10 cm from the end of a 0.5 m long cylinder with 0.5 m diameter. The beam is first computed with dipole in free space. The beam is computed as a function of the angle θ from the axis through the cylinder.

2] In order to approximate the effect of the moon the dipole is then located 10 km above an infinite ground consisting of the regolith of depth, h, over an infinitely deep layer of rock. This configuration is used to take advantage of the Green’s Function (GF) mode in FEKO. In this mode FEKO can only calculate the beam for half plane above the horizon. This mode gives the beam which includes the effect of reflections from the lunar surface on the assumption that the moon is flat. If the antenna is at a distance d above the surface the lunar surface of the moon the surface covers a half angle of \( \sin^{-1}\left(\frac{r}{r+d}\right) \) where r is the radius of the moon.

3] To approximate the net effect of the moon use convolve the beam with the sky using the free space beam over the region not blocked by the moon and use the GF derived beam over the remaining region. In addition, we reduce the sky noise by a factor of 10 and add 200 K when using the GF beam to account from the ~10% reflectivity and emission from the moon.

Figure 1 shows the dipole and its beam in free space. This dipole has a bidirection beam which is almost completely independent of frequency up to 150 MHz. The low frequency limit of about 10 MHz is set by the point at which the sky noise reaching the LNA no longer dominates the receiver noise. Figure 2 shows the antenna beam in the direction normal to the lunar surface for a single frequency. The beam is highly dependent on frequency, beam angle and the height of the antenna above the lunar surface. This requires that FEKO be run over a range of frequency within the spectrometer resolution and also over a range of height of several wavelengths in order to obtain a meaningful average. This requires running FEKO about 25 times to cover a resolution of 400 kHz.
A second method obtaining an estimate of the beam chromaticity effects of the reflections of the sky foreground from the lunar surface is through a scaled version of the geometry. In order to obtain results in a reasonable time the portion of a sphere visible to the antenna is placed up to 40 meters from the antenna. With this geometry a sphere with angular size of 10 degrees (5 degree half angle) can be modelled with FEKO in about 6 minutes per frequency. In addition, the sphere can be covered with a dielectric layer. Figure 3 shows the 5m sphere with dielectric covering. The antenna is 40 m above and above the top of the page. That model confirms that the rms beam chromaticity is proportional to the square of the angular extent of the moon visible from the antenna. Table 1 gives the beam chromaticity results obtained averaging over Galactic hour angle 6-18 hours with 5-physical terms removed over 50-100 MHz.

<table>
<thead>
<tr>
<th>Half angle (deg)</th>
<th>Method 1</th>
<th>Method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>5.0</td>
<td>20</td>
<td>26 (39)</td>
</tr>
</tbody>
</table>

Table 1. Results from small half angles.

A third method of obtaining an estimate is to use the geometric optics shown in Figure 5.

\[
\begin{align*}
  h &= rm \left(\frac{1}{\sin(\text{lim})} - 1\right) \\
  t &= \tan^{-1}\left(\frac{rm \sin(b)}{(rm + h - rm \cos(b))}\right) \\
  i &= b + t \\
  e &= 90 - 2i - t
\end{align*}
\]

where

- lim = half-angle of moon seen from antenna
- h = distance from antenna to surface
- rm = radius of moon
- b = angle from moon center to reflection
- t = angle from antenna to reflection
- i = angle of incidence of reflection
- e = elevation of ray entering reflection point

The reflected power is

\[
\frac{\tan^2(i - r)}{\tan^2(i + r)}
\]

where

\[
r = \sin^{-1}\left(\frac{i}{\varepsilon^{1/2}}\right)
\]

where \(\varepsilon\) is the dielectric of the regolith
it is noted that the polarization of the antenna is such that the electric vector is in the plane of incidence so that the reflection from the regolith is zero at the Brewster angle of incidence of about 64 degrees.

The reflection from the outer surface of the regolith has little, if any frequency dependence and doesn’t contribute significantly to the overall chromaticity. The main source of chromaticity is the presence of a reflection from the rock interface below the regolith. This reflection is delayed by

$$\frac{2d\varepsilon^{\varepsilon/\cos(r)}}{c},$$

where $d = \text{the thickness of the regolith}$, and coherently interferes with the reflection from the regolith. This reflection is also attenuated by the loss in the path through the regolith and is reduced by the reflected power fraction is

$$\tan^2\left(r - r^l\right)/\tan^2\left(r + r^l\right)$$

where $r^l = \sin^{-1}\left(r/\left(\varepsilon_r/\varepsilon\right)^{\varepsilon/2}\right)$

where $\varepsilon_r \sim 8.5$ is the dielectric constant of the rock. The assumed loss in the regolith is 0.2 dB/m which corresponds to $2 \times 10^{-4}$ S/m from $(\sigma/\mu)(\sigma/\varepsilon)^{\varepsilon/2}$.

Table 2 shows the results of the simulations of chromaticity rms with 5-terms removed for 3 cases using the geometric optics approximations. The advantage of this method is that it can be run for larger values of half angle which are not possible using the full EM simulation owing to the large number of wavelengths in the lunar structure. From these results it appears that the antenna could get as close as 960 km before the chromaticity becomes a significant factor. More accurate simulations could be obtained using the far side regolith thickness results from the Change-1 Ya-Qin Jin et al. 41st lunar and planetary science conference 2010.

A plot of the estimated beam chromaticity, with 5-terms removed, is shown for EDGES and case 2 for half angle of 40 degrees.

A study of the various different antenna designs show that the beam chromaticity and S11 are not significantly effected by

1] Changing from a cylinder and sphere to 2 cylinders.
2] Changing from cylinders to cubes so that one can fit inside the other prior to deployment.
3] For a lower frequency coverage version with same 30 cm cube for the receiver and other electronics it is better to deploy long rods from opposite faces of the cube.

The expected performance is given in Table 2 for the antenna dimensions in Figure 5.
<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>Sky noise (K)</th>
<th>ant1 S11 (dB)</th>
<th>ant2 S11 (dB)</th>
<th>ant 1 (K)</th>
<th>ant2 (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$10^6$</td>
<td>-0.001</td>
<td></td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>$3 \times 10^5$</td>
<td>-0.03</td>
<td></td>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>$3 \times 10^5$</td>
<td>-0.002</td>
<td>-6.4</td>
<td>140</td>
<td>$2 \times 10^5$</td>
</tr>
<tr>
<td>30</td>
<td>$3 \times 10^4$</td>
<td>-0.01</td>
<td>-0.5</td>
<td>70</td>
<td>4000</td>
</tr>
<tr>
<td>40</td>
<td>9000</td>
<td>-0.04</td>
<td></td>
<td>80</td>
<td></td>
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<tr>
<td>50</td>
<td>5000</td>
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<td>170</td>
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</tr>
<tr>
<td>100</td>
<td>1000</td>
<td>-1.1</td>
<td></td>
<td>290</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>320</td>
<td>-0.57</td>
<td></td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>300</td>
<td>-0.30</td>
<td></td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Estimate of sky noise entering LNA at each frequency with range of acceptable beam chromaticity. Dimension of antennas are in Figure 5.

The reflection coefficients and signal levels entering the LNA are based on 50 ohms and a perfect 50 ohm match into the LNA. An increase of a factor of 10 or more at the low end can be made by using an LNA with higher input impedance. Adding inductance in series with the antenna can also be used to increase the signal levels. However more study is needed to determine if using matching networks or non standard LNA impedance improves the overall performance over the full frequency range.

Figure 6 shows the estimated beam chromaticity for the EDGES low band antenna, top plot, and the small antenna 960 km above the lunar surface for a 10 m thick regolith in the middle plot and 6 m regolith in the bottom plot. In each case a 5-term polynomial has been removed.
Figure 1. Short dipole in free space.
Figure 2. Beam at 110 MHz using GF mode with antenna 10 km above flat ground.
Figure 3.
Figure 4. Beam facing moon.
Figure 5. Geometric optics and antenna designs under study to cover 20 to 160 and 5-30 MHz.
Figure 6. Beam chromaticity for EDGES and small antenna 960 km above lunar surface for regolith thickness 10 m and 6 m compared with EDGES in top plot.