To: SRT Group  

From: Alan E.E. Rogers and Larry Kimball  

Subject: Measurement of Galactic rotation curve using 21-cm hydrogen emission  

Background material  

1. Radio Astronomy – J.D. Kraus  

Method:  

Use the SRT to observe the H-line spectrum at several points in the plane of the Galaxy. A sample command file is as follows:

```
LST:1800:00:00 / wait until 1800 UST when Galactic center is above  
telescope limits  
: 1420.4 50 / set radiometer center frequency and a number of frequency bins  
: azel 180 45 / point south at 45 degrees  
: calibrate / calibrate radiometer  
: Galactic 0 0 / move telescope to Galactic center  
: record g00.rad / start data file  
: 600 / take data for 600 seconds  
: roff / turn off data recording  
: Galactic 10 0 / move telescope to Galactic longitude of 10 degrees  
: record g10.rad  

/repeat for longitudes to 90 degrees
```

This suggested observing schedule provides spectra for 10 points in the Galactic plane. These spectra should be plotted and the velocity of the most positive velocity emission measured relative to the local standard of rest (i.e. after removal of the Doppler shift due to the local motion of the Sun and the Earth). This can be accomplished by running the java program “splot” on the data files, doing your own analysis on the data files or by being present during the observations to make notes or capturing the screen.

The velocity V of a frequency channel is given

\[ V = \frac{(1420.406-f) V_c}{1420.406 - V_{lsr}} \text{ km/s} \]
Where \( V_c = \) velocity of light 299790 km/s
\( V_{\text{lsr}} = \) velocity of observer relative to local standard of rest
\( f = \) frequency of channel = \( f_{\text{L.O.}} + 0.04 \)

The real time control program and splot both contain a routine which calculates the \( V_{\text{lsr}} \) and splot labels the spectrum axis with velocity relative to the local standard of rest.

**Sample results:**

<table>
<thead>
<tr>
<th>Galactic longitude (deg)</th>
<th>most negative Freq. (MHz)</th>
<th>max velocity km/s</th>
<th>estimated error km/s</th>
<th>tangential distance kpc</th>
<th>( \omega_o R ) km/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1419.60</td>
<td>150</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>1419.50</td>
<td>180</td>
<td>-</td>
<td>1.5</td>
<td>39</td>
</tr>
<tr>
<td>20</td>
<td>1419.70</td>
<td>130</td>
<td>10</td>
<td>2.9</td>
<td>75</td>
</tr>
<tr>
<td>30</td>
<td>1419.82</td>
<td>120</td>
<td>10</td>
<td>4.2</td>
<td>109</td>
</tr>
<tr>
<td>40</td>
<td>1419.98</td>
<td>95</td>
<td>10</td>
<td>5.5</td>
<td>142</td>
</tr>
<tr>
<td>50</td>
<td>1420.10</td>
<td>70</td>
<td>10</td>
<td>6.5</td>
<td>168</td>
</tr>
<tr>
<td>60</td>
<td>1420.18</td>
<td>60</td>
<td>10</td>
<td>7.4</td>
<td>192</td>
</tr>
<tr>
<td>70</td>
<td>1420.34</td>
<td>30</td>
<td>10</td>
<td>8.0</td>
<td>207</td>
</tr>
<tr>
<td>80</td>
<td>NOT OBSERVED</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>1420.42</td>
<td>10</td>
<td>10</td>
<td>8.5</td>
<td>220</td>
</tr>
</tbody>
</table>

**Sample spectrum:**
Theory:

Model of the Galactic rotation

We model the Galactic motion as circular motion with monotonically decreasing angular rate with distance from the center. For example if the mass of the Galaxy is all at the center the angular velocity at distance $R$ is given by

$$\omega = M^{-\gamma}G^{1/2}R^{-3/2}$$

where $M =$ central mass kg
$G =$ gravitational constant $6.67 \times 10^{-11} \text{Nm}^2\text{kg}^{-2}$
$R =$ distance from central mass m

If we are looking through the Galaxy at an angle $\gamma$ from the center the velocity of the gas at radius $R$ projected along the line of site minus the velocity of the sun projected on the same line is

$$V = \omega R \sin \delta - \omega_0 R_0 \sin \gamma$$

as illustrated in figure below

![Diagram](https://example.com/diagram.png)

where $\omega =$ angular velocity at distance $R$
$\omega_0 =$ angular velocity at a distance $R_0$
$R_0 =$ distance to the Galactic center
$\gamma =$ Galactic longitude
Using trigonometric identity

\[ \sin \delta = R_0 \sin \gamma / R \]  

(2)

and substituting into equation (1)

\[ V = (\omega - \omega_0) R_0 \sin \gamma \]  

(3)

(which is equation 8-51 in Kraus)

The maximum velocity occurs where the line of site is tangential to the circular motion in which case

\[ R = \sin \gamma R_0 \]  

(4)

And hence

\[ V_{\text{max}} = \omega R = V_{\text{max}}(R) + \omega_0 R \]  

(5)

The Sun’s rotational velocity \( \omega_0 R_0 \) and distance to the Galactic center can be taken from other measurements to have values of

\[ \omega_0 R_0 = 220 \text{ km/s} \]
\[ R_0 = 8.5 \text{ kpc} = 2.6 \times 10^{17} \text{ km} \]

Data analysis:

\[ V_{\text{max, observed}}(R) + \omega_0 R \]  

(6)

is plotted using the values of \( V_{\text{max, observed}} \) and the calculated values of \( \omega_0 R \) from table of sample results:
Interpretation of results:

The rotational velocity doesn’t depend strongly on the distance from the center at least for $R > 3$ kpc. [Measurements close to the Galactic center are difficult and the integration needs to be sufficient to reduce the noise so that the maximum velocity edge of the emission can be determined.] If the motion was entirely Keplerian (i.e. classical circular orbits around a central mass) the rotational velocity would decrease with a $R^{-1/2}$ dependence. The lack of decline with distance suggests that not all the mass is in the center of the Galaxy.

Exercises for the student:

1. What Galactic mass distribution might fit your data?

2. On the basis of Keplerian motion what is an approximate estimate of the total mass of the Galaxy in solar masses ($M_{\odot} \approx 2 \times 10^{30}$ kg)?

3. Include the Galactic center in your observations. What can you conclude from the observed spectrum?

4. Include observations at negative Galactic longitude and longitudes greater than 90.

5. Observe the H-line spectrum at longitude 180 degrees. What would you expect to observe on the basis of the simple model of circular motion? What would the H-line spectra look like if the Galaxy rotated as a rigid body?

6. Observe the H-line several points crossing the Galactic plane. Is the H-emission from a thin disk? Or can you resolve the “thickness” of the plane given the resolution of the SRT.

7. How do you explain the sudden transition from the lack of velocity spread in the H-line at Galactic longitude less than 20 degrees to a double peaked spectrum at longitudes greater than 20 degrees?

8. How can you use your Galactic velocity dependence with distance to map the distribution of hydrogen?

The material in the introduction was provided by Maria Stefanis, Mount Holyoke and Larry Kimball, University of Massachusetts Lowell.

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1 Ans: Somewhere between large central mass and a uniform disk
2 Ans: $\approx 10^{11} M_{\odot}$
3 Ans: We started to see evidence that the hydrogen is concentrated in “arms” of the Galaxy.
4 Ans: Use the intensity to determine the density of Hydrogen and velocity to determine its distance from the center being aware of the ambiguity in Galactocentric Longitude.