To: SRT Group
From: Alan E.E. Rogers

Subject: Interferometer geometry calculations

For the “VLBI” mode we start with the latitude, longitude and height of each end of the “baseline” and convert to geocentric right handed x, y, z coordinates. This coordinate conversion is done by function

\[
x = (n + hgt) \cos(lat) \cos(lon)
\]
\[
y = (n + hgt) \cos(lat) \sin(lon)
\]
\[
z = (n + (1-e) + hgt) \sin(lat)
\]

where

\[
n = \frac{a}{(1-e\sin^2(lat))^{1/2}}
\]
\[
a = 6378137 \text{ m} \quad \text{WGS84}
\]
\[
e = 2f-f^2
\]
\[
f = 1/298.257223563 \quad \text{WGS84}
\]

The vector baseline is defined as the vector from site1 (the “reference” site) to site2 (the remote site)

\[
b_x = x_2 - x_1
\]
\[
b_y = y_2 - y_1
\]
\[
b_z = z_2 - z_1
\]

The delay \( \tau \) of a signal’s arrival at the remote site is

\[
\tau = -\frac{\vec{b} \cdot \vec{s}}{c} = -\left( b_x s_x + b_y s_y + b_z s_z \right)/c
\]

Where \( c \) = velocity of propagation

\( \vec{s} = \) unit vector in the direction of the source

\( s_x = \cos(\text{dec}) \cos(\text{gha}) \)
\( s_y = -\cos(\text{dec}) \sin(\text{gha}) \)
\( s_z = \sin(\text{dec}) \)

where \( \text{gha} = \text{gst} - \text{ra} \) = Greenwich hour angle
\( \text{gst} = \) Greenwich sidereal time
\( \text{ra} = \text{apparent right ascension} \)
\( \text{dec} = \text{apparent declination} \)

or from the derivatives of the phase with respect to \( \text{ra} \) and \( \text{dec} \)

\[
\phi = \left( \frac{2 \pi}{\lambda} \right) \left( \cos(\text{dec}) \cos(\text{gha}) b_x - \cos(\text{dec}) \sin(\text{gha}) b_y + \sin(\text{dec}) b_z \right)
\]

In units of fringes per arc second

\[
\begin{align*}
\mu &= (b_x \sin(\text{gha}) + b_y \cos(\text{gha})) \left( \frac{\pi}{648,000 \lambda} \right) \\
\nu &= (b_x \cos(\text{dec}) - b_y \cos(\text{gha}) \sin(\text{dec}) + b_y \sin(\text{gha}) \sin(\text{dec}) \left( \frac{\pi}{648,000 \lambda} \right))
\end{align*}
\]

The interferometer phase (normally defined as being positive (NRAO’s convention) when the signal arrives earlier at the 2\(^{\text{nd}}\) site is

\[
\phi = \frac{+2 \pi \vec{b} \cdot \vec{s}}{\lambda} \quad (\text{radians})
\]

or \( \phi = -2 \pi \frac{f}{\lambda} \quad (\text{radians}) \)

where \( \lambda = \text{wavelength (m)} \)
\( f = \text{frequency (Hz)} \)

The components of the baseline projected in the direction of the source in the directions of increasing RA and increasing declination are known as \( u \) and \( v \) and are often expressed in units of fringes per arc second. These can be derived from the baseline projections

\[
\begin{align*}
\mu &= b_x \sin(\text{gha}) + b_y \cos(\text{gha}) \\
\nu &= b_x \cos(\text{dec}) - b_y \cos(\text{gha}) \sin(\text{dec}) + b_y \sin(\text{gha}) \sin(\text{dec})
\end{align*}
\]