To: UVLBI Group  
From: A.E.E. Rogers  
Subject: Detection threshold for bispectral fringe searches  

In searching for weak fringes using the bispectrum it is advantageous to use a 2 step process unless the closure phase is known in advance. In our paper (Rogers, Doeleman & Moran, A.J. 109, 1391, 1995) we give the algorithm in equ (74) but do not discuss the detection threshold.

If the known the optimum search algorithm is to search for a maximum in the real part of

\[ \sum_{i=1}^{M} \text{amp}_i \cos(\theta_i - \theta_c) \]

where \( \text{amp}_i \) is the magnitude of the triple product for the \( i \)th segment and \( \theta_c \) is the assumed closure phase. If the closure phase is unknown the search can be made using a 2 step process:

First determine the closure phase by computing

\[ R = \sum \text{amp}_i \cos(\theta_i) \]
\[ I = \sum \text{amp}_i \sin(\theta_i) \]
\[ \theta_c = a \tan2(I, R) \]
\[ \sum \text{amp}_i \cos(\theta_i - \theta_c) \]

For a single segment this equivalent to finding the maximum magnitude which, as discussed in Rogers et al. degrades the SNR by \( \sqrt{2} \). However for a large number of segments there is only a small degradation in SNR over the case of assuming an apriori value for \( \theta_c \) and in the case of a large number of segments the SNR is approximated by

\[ \text{SNR} = \frac{\text{amp}_i \cos(\theta_i - \theta_c)}{(\text{amp}_i^2 \sin^2(\theta_i - \theta_c))^{1/2}} \]

simulations made with

\[ s_1 = 3 \]
\[ s_2 = 1 \]
\[ s_3 = 0.75 \]
<table>
<thead>
<tr>
<th>#segments</th>
<th>( s_3 )</th>
<th>SNR(_1)</th>
<th>SNR(_2)</th>
<th>SNR(_3)</th>
<th>SNR(_4)</th>
<th>SNR(_5)</th>
<th>R1</th>
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<td>0.75</td>
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<td>3.09</td>
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<td>3.31</td>
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<tr>
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<td>5.03</td>
<td>4.30</td>
<td>5.21</td>
<td>5.34</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Where SNR\(_1\) = calculated SNR using equ (49)
   SNR\(_2\) = using equ (74) assuming an apriori closure phase
   SNR\(_3\) = max SNR\(_2\) in search of \(4\times10^4\) trials without signal
   SNR\(_4\) = using equ (74) estimated closure
   SNR\(_5\) = max SNR\(_2\)

The ratio \( R = (\text{SNR}_4/\text{SNR}_5)/(\text{SNR}_4/\text{SNR}_2) \) is a measurement of the equivalent loss of sensitivity for low level detections which results from having to determine the closure phase from the data itself. The loss is in the range of 5 to 16 percent depending on the segment SNRs and the number of segments.