BBD Memo #034

Massachusetts Institute of Technology
Haystack Observatory
WESTFORD, MASSACHUSETTS 01886

DATE: 04-23-2009

To: Broadband Development Group

From: C. J. Beaudoin

Subject: Phase Cal Amplitude RF Frequency Dependence

Purpose: To define the RF frequency at which phase cal amplitudes should be set given the measured frequency dependence of the phase cal signal's spectrum. If phase cal levels are set at a low frequency , the phase cal amplitudes at the highest frequency will be too weak. In contrast, if the phase cal levels are set a the highest frequency , the phase cal amplitudes at the lowest frequency may be too strong. This note attempts to identify the point at which a compromise between both cases can be met.

Because the new broadband system utilizes the same set of hardware to receive a very broadband range of frequencies, the phase cal amplitudes for each frequency band (i.e. each UDC band) cannot be adjusted independently. This system architecture is in contrast to the S/X system which utilizes separate S and X band receivers, thereby allowing the phase cal signal levels to be adjusted independently in each frequency band. Given this constraint of the broadband system, we are faced with having to deal with phase cal amplitudes that will varying from UDC band to UDC band (unless of course the UDC are detecting the same band). In order to characterize the extent to which the phase cal amplitudes will vary across the current operating band (3GHz – 10 GHz) the phase cal rail power spectrum was measured. Figure 1 displays the measured phase cal spectrum along with a quadratic fit to the measured data.

It is possible to analyze how much the phase cal amplitude will vary given the power spectrum shown in Figure 1. The phase cal amplitude (A_{pc}) as derived in BBD memo #31, is related to the total phase cal power (P_{pc}), the number of rails (N_r), and the receiver noise power (P_n) (all defined within a 32 MHz channel) by the following:

$$A_{pc} = 1000\sqrt{\frac{\eta}{\eta+1}\frac{1}{N_r}} \qquad \text{Eq (1)}$$

$$\eta = \frac{P_{pc}}{P_n}$$

In Eq (1) a term of ‘1/2’ was dropped as it is insignificant in comparison to the amplitudes in practice and becomes bothersome to carry through the algebra. For the purpose of this note, I make the assumption that P_n is constant over the 3-10 GHz band in order to facilitate the analysis. Making this assumption, P_n has no impact on the analysis since we are only interested in how variations of the phase cal spectrum induce variations in the phase cal amplitude. This being the case, P_n can be arbitrarily set to “1” such that Eq (1) becomes:

$$A_{pc} = 1000\sqrt{\frac{P_{pc}(f)}{P_{pc}(f)+1}\frac{1}{N_r}} \qquad \text{Eq(2)}$$

The parenthetic f represents RF frequency as indicated in figure 1.
By setting the phase cal power levels, one is introducing more or less attenuation of the phase cal signal in order to achieve a target phase cal amplitude. The attenuation α_s is set in order to obtain the target phase cal amplitude A_{pc}^t at the target frequency f_t:

$$A_{pc}^t = 1000\sqrt{\frac{\alpha_s P_{pc}(f_t)}{\alpha_s P_{pc}(f_t)+1}\frac{1}{N_r}} \qquad \text{Eq(3)}$$

Rearranging Eq (3) the required attenuation needed to achieve the target phase cal amplitude at the target RF frequency f_t is:

$$\alpha_s = \frac{N_r\left(\frac{A_{pc}^t}{1000}\right)^2}{\left(1-N_r\left(\frac{A_{pc}^t}{1000}\right)^2\right)P_{pc}(f_t)} \qquad \text{Eq(4)}$$

Given the attenuation value as calculated using Eq (4), one can observe how the phase cal amplitudes will vary across the band if we set the attenuation at target frequency f_t. To do so, the following equation is used:

$$A_{pc} = 1000\sqrt{\frac{\alpha_s P_{pc}(f)}{\alpha_s P_{pc}(f)+1}\frac{1}{N_r}} \qquad \text{Eq(5)}$$

where $A_{pc} = A_{pc}^t$ when $f = f_t$. Figure 2 displays bounds on the phase cal amplitude A_{pc} over the 3-10 GHz target frequency f_t when $A_{pc}^t = 40$ and $N_r = 6$; the quadratic fit for the phase cal spectrum shown in figure 1 is substituted for $P_{pc}(f)$. The red and blue curves are the phase cal amplitudes at 3 GHz and 10 GHz when the phase cal level is set at the target frequency f_t indicated on the horizontal axis. For example, at f_t = 3 GHz the 3 GHz phase cal amplitude is equal to the target value 40 and similarly at f_t = 10 GHz the 10 GHz phase cal amplitude is also equal to the target value. Equations (19),(20) in BBD memo #32 provides a relation between the phase cal amplitude and SNR. In order to obtain phase cal phase estimates to be better than 1° with 95% confidence requires a power SNR of 41 dB (see figure 1 in BBD memo #32). Given this SNR, a phase cal extraction period of 2 seconds, a channel bandwidth of 32 MHz and Nr = 6, the minimum phase cal amplitude required to achieve such phase precision is A_{pc} = 14. There has also been problems in the past with the broadband system phase cal levels being too strong which ultimately led to erroneous phase cal estimates; this level was approximately 150. Normal phase cal amplitudes, however, are generally in the range of 10 – 60. If we utilize 60 as an upper bound and 14 as a lower bound figure 2 indicates that if the phase cal levels are set at approximately 6.5 GHz using $A_{pc}^t = 40$, the phase cal amplitudes will become no larger than 60 and always be above 14.

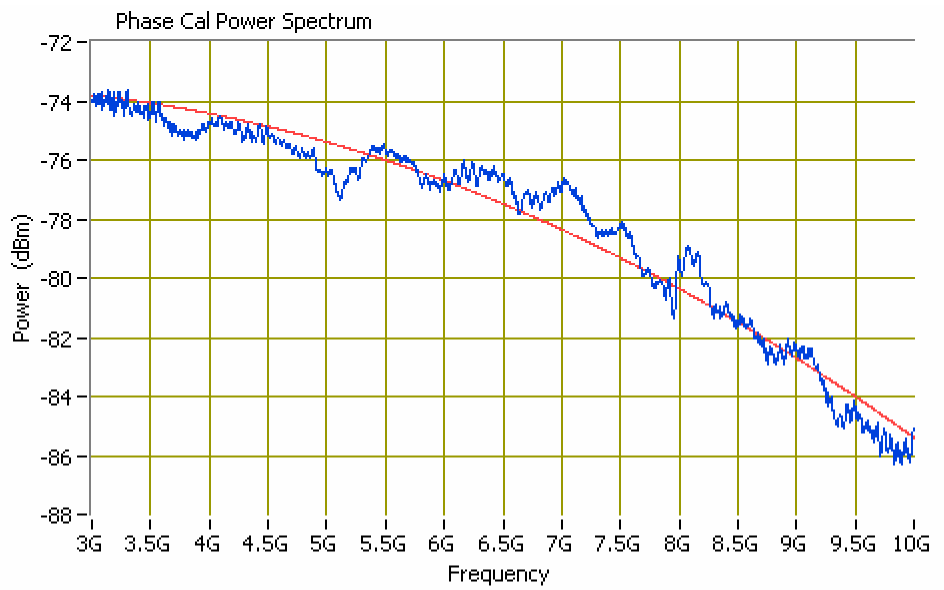

Figure 1: Measured phase cal spectrum (blue) with quadratic fit (red)

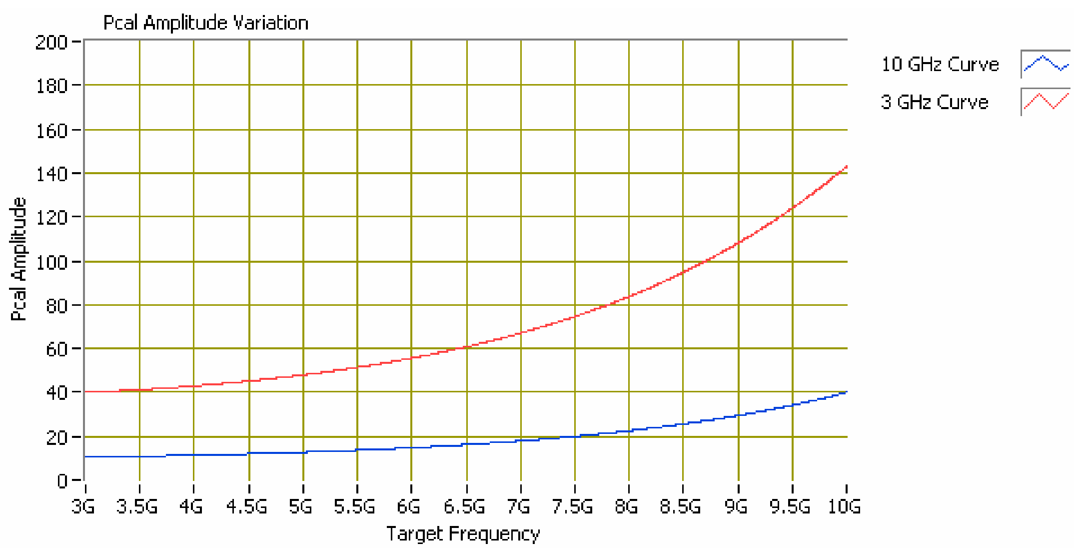

Figure 2: Phase cal amplitudes at 3 and 10 GHz as a function of target frequency. The target phase cal amplitude is 40.