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To: Broadband Development Group

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Subject: Computation of saturation overhead improvement in active stages with incorporation of broadband equalization

Background

The incremental noise temperature contribution of a noisy stage in the RF signal path is based on the strength of the signal presented at the input of the device relative to the thermal noise inherent in the device itself. For narrowband devices, the assumption of near-constant noise figure over the operational bandwidth of the device is generally valid. However, this is assumption may not be considered valid for ultra broadband microwave devices. In this case, the required driving point signal power level (to avoid significant incremental noise temperature) can be established based upon the worst-case noise figure of the device over the bandwidth of the device in question. Assuming the frequency dependence in the noise figure of the device is largely linear (as is the case with RF-over-fiber links in the range 2-14 GHz), this note will show that establishing the driving point signal level based upon the maximum noise figure is not optimal in consideration of saturation overhead.

Derivation

Noise figure model: $NF(f) = NF_{\min} + \frac{\Delta NF}{B}(f - f_o)$

NF(f): frequency-dependent noise figure of the device
B: device bandwidth
f_o: device low frequency specification
NF_{min}: minimum noise figure of the device over the device bandwidth
ΔNF: change in noise figure over the specified bandwidth

NFmin assumed to be greater than 20dB so that 10log10(10^(NF/10)-1) ~ NF

Unequalized Total Power (in dB) relative to *293k*:

$$P_{ueq}^{dB} = NF_{\max} + SNR + 10\log_{10}(B) = NF_{\min} + \Delta NF + SNR + 10\log_{10}(B)$$

k: Boltzmann's constant
SNR: strength of input signal relative to device's thermal noise

Equalized PSD relative to *293k*:

$$PSD_{eq}^{dB} = NF_{\min} + \frac{\Delta NF}{B}\left(f - f_o\right) + SNR$$

$$PSD_{eq} = 10^{\frac{NF_{\min}+SNR}{10}} 10^{\frac{\Delta NF}{10B}\left(f-f_o\right)}$$

Integrate equalized PSD to compute total power

$$P_{eq} = 10^{\frac{NF_{\min}+SNR}{10}} \int_{f_o}^{f_1} 10^{\frac{\Delta NF}{10B}\left(f-f_o\right)} df = 10^{\frac{NF_{\min}+SNR}{10}} \left(\frac{10B}{\Delta NF \log_e(10)}\right)\left(10^{\frac{\Delta NF}{10B}\left(f-f_o\right)}\Big|_{f_o}^{f_1}\right)$$

$$P_{eq} = 10^{\frac{NF_{\min}+SNR}{10}} \left(\frac{10B}{\Delta NF \log_e(10)}\right)\left(10^{\frac{\Delta NF}{10}} - 1\right) = 10^{\frac{NF_{\min}+SNR}{10}} \left(\frac{10B}{\log_e(10)}\right)\left(\frac{10^{\frac{\Delta NF}{10}} - 1}{\Delta NF}\right)$$

where the substitution $B = f_1\text{-}f_o$ was implemented above

Express P_{eq} in dB

$$P_{eq}^{dB} = NF_{\min} + SNR + 6.3778 + 10\log_{10}(B) + 10\log_{10}\left(\frac{10^{\frac{\Delta NF}{10}} - 1}{\Delta NF}\right)$$

Improvement in overhead is the difference between the relative unequalized and equalized total power levels:

$$\Delta OH = P_{ueq}^{dB} - P_{eq}^{dB} = NF_{\min} + SNR + 10\log_{10}(B) + \Delta NF$$
$$- \left[NF_{\min} + SNR + 10\log_{10}(B) + 6.3778 + 10\log_{10}\left(\frac{10^{\frac{\Delta NF}{10}} - 1}{\Delta NF}\right)\right]$$

$$\Delta OH = \Delta NF - 6.3778 - 10\log_{10}\left(\frac{10^{\frac{\Delta NF}{10}} - 1}{\Delta NF}\right)$$

It is worth noting that $\lim_{\Delta NF \to 0} \Delta OH = 0$ which indicates that there is no improvement in the saturation overhead when there is no change in the noise figure over the operational bandwidth. It is also worth noting that the improvement in saturation overhead is independent of the SNR.

The expression for ΔOH above reveals that a 4 dB increase in saturation overhead can be recouped if the variation in noise figure is 10 dB and a 5 dB improvement can be gained if the noise figure variation is 13 dB. Figure 1 displays the noise figure variation of two Photonics Systems Inc. RF-over-fiber links.

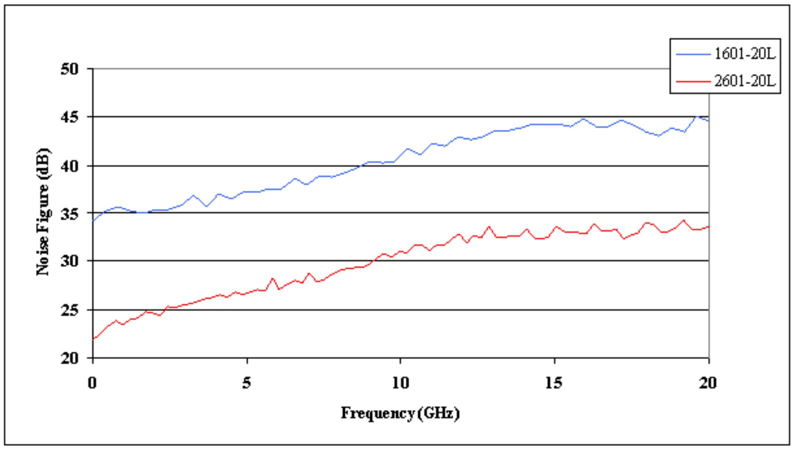

Figure 1 Typical noise figure data of two Photonics Systems Inc. RF-over-fiber links