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July 9, 2007

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To: VSRT Group  
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 Subject: A method for detecting active solar regions at 12 GHz

### Introduction

At 12 GHz the Sun is usually close to a uniformly bright disk. While a single baseline VSRT interferometer can observe the Sun it is hard to detect changes in the Sun because the fringe strength depends on the antenna pointing, amplifier and detector gains, as well as atmospheric attenuation. Phase information on a single baseline is not useful because the LNB have unsynchronized free running local oscillators. To circumvent these difficulties we can make a 3 antenna (3 baseline) interferometer to observe the closure phase (see memo #3 for details of the closure phase concept). Further a special baseline configuration in which the longest baseline resolves the uniform solar disk and lies at the first null in the visibility is especially useful. We shall show that in this case the closure phase is equal to the phase coherent interferometer phase “phase referenced” to the center of the solar disk when the deviations of the solar image from a uniform disk are small.

### Visibility of the uniform disk

The visibility of the uniform disk is given by the following 2-D integral

$$\begin{aligned} V(z) &= \int_0^R \int_0^{2\pi} e^{irz \cos \theta} r d\theta dr / (\pi R^2) \\ &= \int_0^R 2r J_0(rz) dr / R^2 \\ &= 2 J_1(Rz) / (Rz) \end{aligned}$$

where  $z$  = interferometric phase in radians per radian

=  $2\pi \times$  projected baseline/wavelength

$R$  = radius of solar disk in radians.

$J_0$  = zero order Bessel function

$J_1$  = first order Bessel function

The visibility is also given by the 1-D integral from simple geometry of a circle

$$V(z) = \int_0^R (R^2 - r^2)^{1/2} \cos(rz) dr / (\pi R^2 / 4) = 2 J_1(Rz) / (Rz)$$

## Visibility of non-uniform solar disk

The complex fringe amplitude,  $A(z)$  is additive so that

$$A(z) = pV_D(z) + aV_s(z)$$

where  $V_D$  = complex visibility of uniform disk

$V_s$  = complex visibility of the solar disk with uniform disk subtracted

$p$  = power in the uniform disk

$a$  = power in the difference

If the longest projected baseline is in the first null of  $J_1(Rz)/(Rz)$  then  $A(z) = aV_s(z)$  on the longest baseline and if  $(a/p) \ll 1$  then  $A(z) \approx pV_D(z)$  on the shorter baselines.

Since  $V_D(z)$  is real and positive on the shorter baselines the visibility phases on these baselines are zero and hence the closure phase,  $\phi_c$  is

$$\phi_c = \text{atan2}(\text{Im}V_s, \text{Re}V_s).$$

This analysis is only valid when the longest baseline is in the first null of the visibility so that an empirical modeled is needed for other baseline configurations.

Figure 1 shows an example of the calculated amplitudes and closure phase for an amplitude and closure phase for an interferometer with two dishes on the sample az/el mount and the third dish at a distance of 9.7 feet away along an azimuth of 106 degrees. The red curves are for a uniform Sun on day 163 and the blue curves for a uniform Sun plus a sunspot having only 2% of the total solar flux. Figure 2 shows the simulation for a range of sunspot strength from 2% to 10%. Figure 3 show the simulation for a sunspot of 0.5% flux moved from a position 16% offset center to 80% off center.

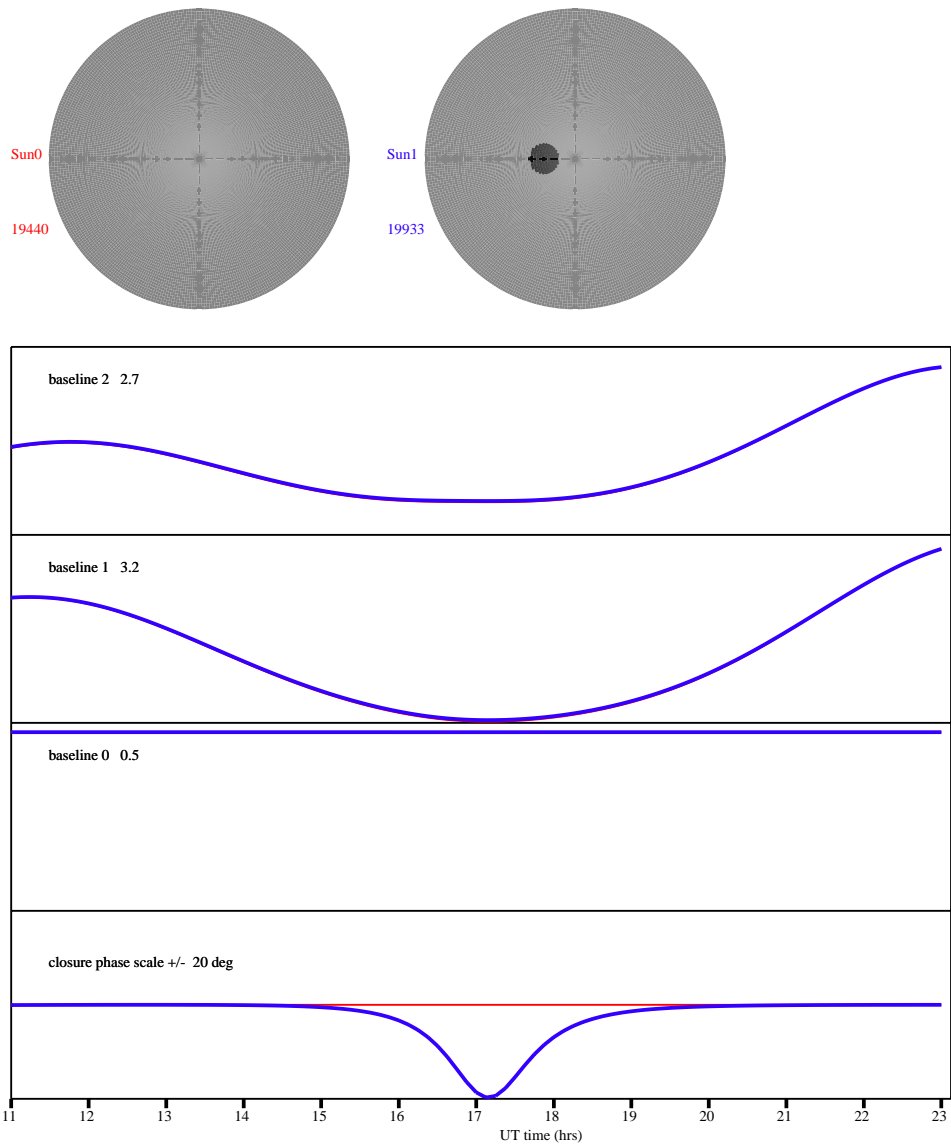


Figure 1. Simulation of amplitude on each baseline and the closure phase. The order of the plots, from the top, is intermediate, longest, shortest and closure phase. The blue curves are for the uniform Sun plus a sunspot.

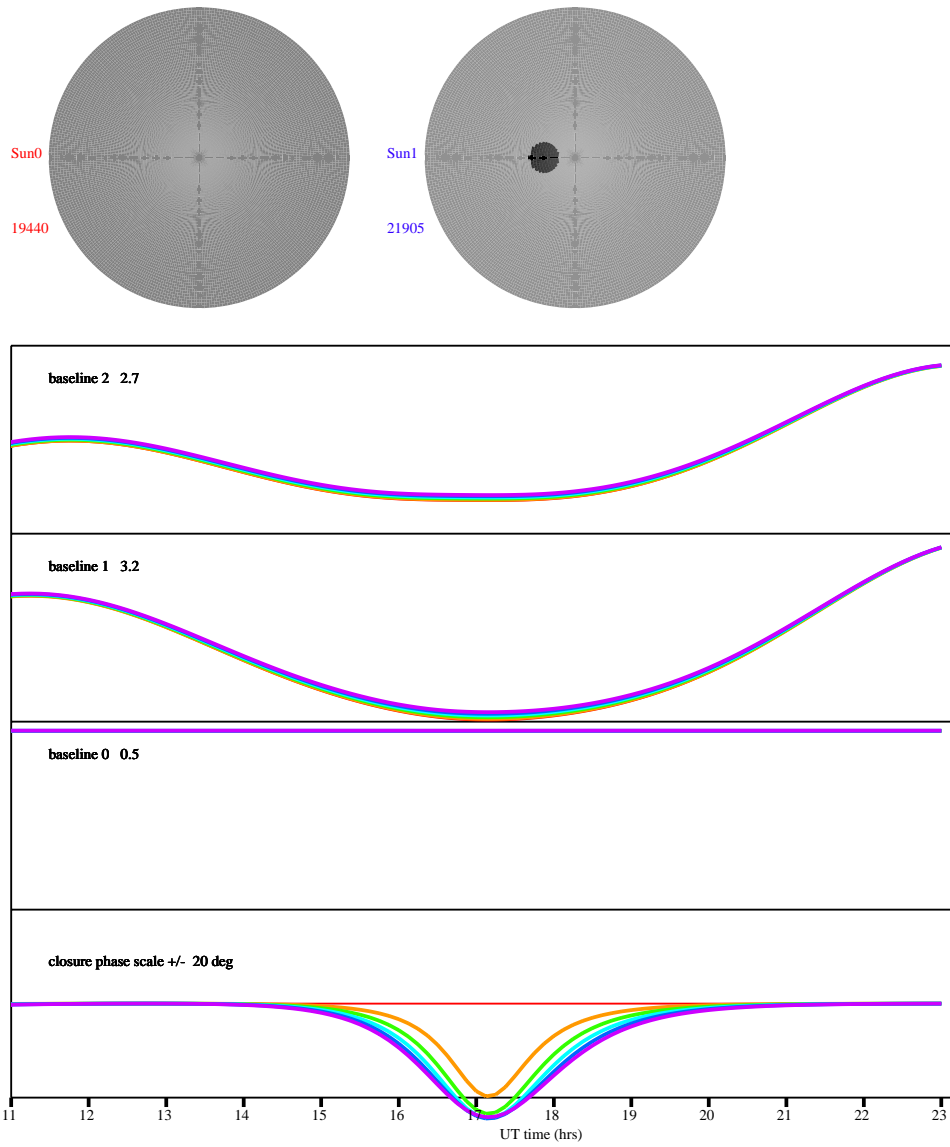


Figure 2. Effect of changing the sunspot intensity.

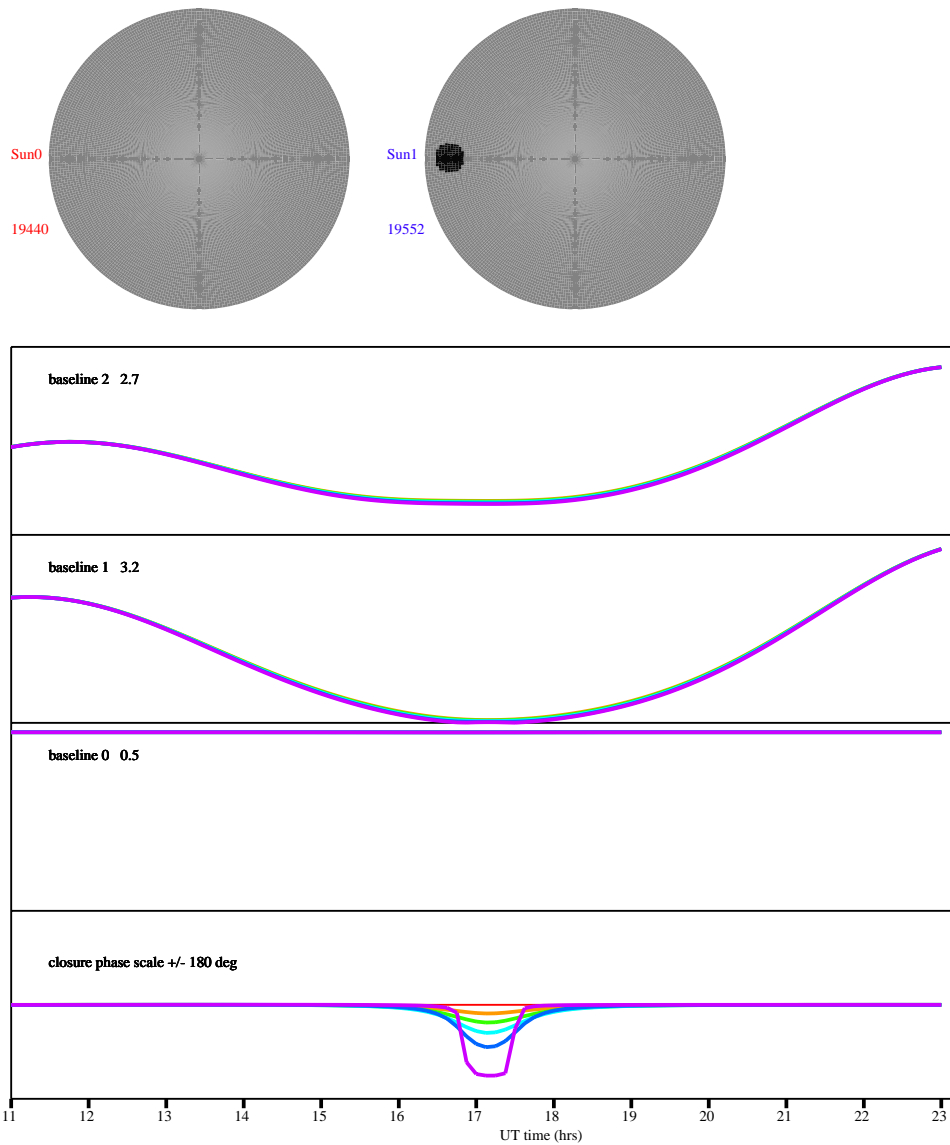


Figure 3. Effect of changing the sunspot position in the E-W direction.