MASSACHUSETTS INSTITUTE OF TECHNOLOGY
HAYSTACK OBSERVATORY

WESTFORD, MASSACHUSETTS 01886

February 23, 2009

Telephone: 781-981-5407
Fax: 781-981-0590

To: VSRT Group
From: Alan E.E. Rogers
Subject: Simplified radiation balance needed to explain Global warming and upper atmospheric cooling.

1] Planet without an atmosphere
Consider a planet without an atmosphere. In this case the energy from the Sun is balanced by the radiation from the planet's surface out into space.

$$S\left(1-A\right)\pi r^2 = \sigma T^4 4\pi r^2 \qquad (1)$$

Where S = solar flux[1] = 1370 watts/m^2
r = radius of the Earth
A = Earth's "albedo" fraction of solar energy reflected ~ 0.367
σ = Stefan-Boltzmann constant
= 5.67 ×10^{-8} watts/m^2/K^4

πr^2 is the cross-section area of the plane wave rays from the Sun intercepted by the Earth. $4\pi r^2$ is the total surface area of the Earth from which the Earth's heat is lost into space going out radially in all directions. σT^4 is known as the Stefan-Boltzmann law and is the integral of the black body radiation given by Planck's law

$$B = \frac{2h\nu^3}{c^2}\frac{1}{\left(e^{h\nu/KT}-1\right)} \qquad (2)$$

over all frequencies (or equivalently all wavelengths) and all directions.
B = watts/m^2/solid angle/Hz
h = Planck constant
ν = Frequency Hz
c = velocity of light
K = Boltzmann's constant

If we solve equation (1) we get

$$T^4 = S\left(1-A\right)/\left(4\sigma\right) \qquad (3)$$

T = 249 K = -24 C
A temperature well below the Earth's average temperature.

[1] The average over the sphere normal to the surface is 1370/4=342 W/m^2

2] Adding a "blanket" warms the Earth

Now if we add an atmosphere which absorbs some of the radiation leaving the Earth, while hardly absorbing any of the Sun's radiation the Earth will warm up to maintain the energy balance. Constituents of the atmosphere, like carbon dioxide, are known as "greenhouse" gases because they have spectral lines which absorb in the infrared part of the spectrum (see figure 1) where the radiation from the Earth dominates while they are quite transparent in the visible part of the spectrum where the Sun's energy is concentrated. These "greenhouse" gases warm the Earth in much the same way as a "blanket" keeps you warm in bed at night.

3] In warming the Earth the upper atmosphere cools

To look in more detail at what happens when we add a greenhouse gas we need to consider the "Radiative transfer" in a layer of the atmosphere containing the region where the infrared is absorbed. When a gas contains spectral lines these lines not only absorb radiation but also emit radiation in the same frequency band. If a wave travels through an absorbing gas the power in this wave decreases exponentially. That is the power is multiplied by the factor $e^{-\tau}$, where τ is the opacity of the gas. Equivalently the fraction of power lost is $\left(1-e^{-\tau}\right)$. However, this "lost" power is "compensated" by emission from the gas which is proportional to $\left(1-e^{-\tau}\right)$.

If the temperature of the gas is the same as the temperature of the Earth's surface then the energy lost by the radiation leaving the Earth is exactly compensated by an equal amount of energy from the gas going out into space. The emission from the layer in the atmosphere goes out in both directions, up and down, as illustrated in Figure 2.

Now rewriting the energy balance (as illustrated in Figure 2) equations for 3 cases removing the common factor $4\pi r^2$:

A] Above the atmosphere

$$\frac{S\left(1-A\right)}{4} = \left(1-f\right)\sigma T_e^4 + f\sigma T_a^4 \tag{4}$$

where T_e = Earth surface temperature

T_a = atmosphere temperature

f = fraction absorbed in atmosphere

= $\left(1-e^{-\tau}\right)$

B] In the atmosphere

$$f\sigma T_e^4 + q = 2f\sigma T_a^4 \tag{5}$$

where q is the power absorbed by a layer in the atmosphere in the ultraviolet frequency range. q is very small in the troposphere but is significant in the stratosphere and mesosphere where O_2 and O_3 have spectral lines in the uv.

C] Below the atmosphere

$$\frac{S(1-A)}{4} + f\sigma T_a^4 - q = \sigma T_e^4 \tag{6}$$

Note: Equation 6 can also be derived from equations 4 and 5 as these 3 equations are not independent. Solving the equations for T_e and T_a we get

$$T_e = \left[\frac{S(1-A) - 2q}{4\sigma(1 - f/2)}\right]^{1/4} \tag{7}$$

$$T_a = \left[\frac{T_e^4 + q/(f\sigma)}{2}\right]^{1/4} \tag{8}$$

some results are as follows

f	q (w)	T_e (K)	T_a (K)
0	0	249	-
0.6	0	272	229
0.8	0	283	238
0	50	241	-
0.1	50	244	280
0.2	50	251	242

The results for $q = 0$ are relevant to the troposphere and show that increased opacity results in warming of both the Earth and the atmosphere. The results for $q = 50$ w are appropriate for a layer much higher in the atmosphere. In this case T_e should be considered as the lower atmosphere temperature. In this case the upper atmosphere temperature T_a, decreases with increased opacity.

In reality the energy budget involves the additional processes as illustrated in Figure 3.

This simplified analysis makes the following simplifications:

1] Only a single uniform layer is considered. A more accurate model should consider many layers of different temperatures.

2] Only radiative transfer is considered. This is only a good approximation for the upper layers of the atmosphere for which circulation is less important.

3] The power is assumed to be proportional to T^4. An accurate model needs to take into account the actual absorption line shapes and frequencies. However it is a good assumption that the power emitted will be a monotonically increasing function of temperature as in equation (2) which is all that is required to show that the Earth will warm while the upper atmosphere cools in the case of $q > 0$.

In addition the changes in temperature which are expected to result from changes in opacity of the greenhouse gases need to take into account "feedback" processes. For example more CO_2 may also result in the melting of snow and ice which in turn will reduce the albedo of the Earth resulting in a further increase in the average temperature. The details of all these "feedback processes" along with a more accurate model of global warming is given in "The Physics of Atmosphere," 3rd Edition by John Houghton, published by the Cambridge University Press, 2002.

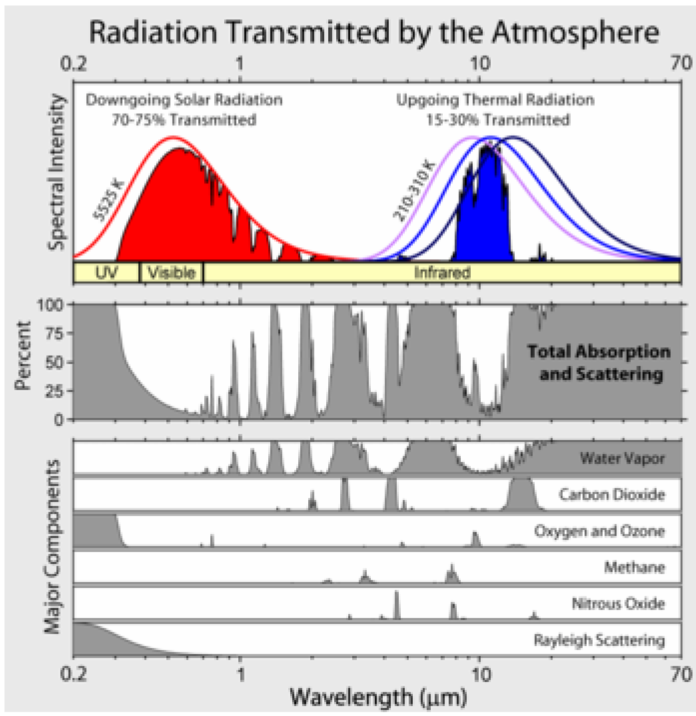

Figure 1. Radiation spectrum transmitted by the Earth's atmosphere

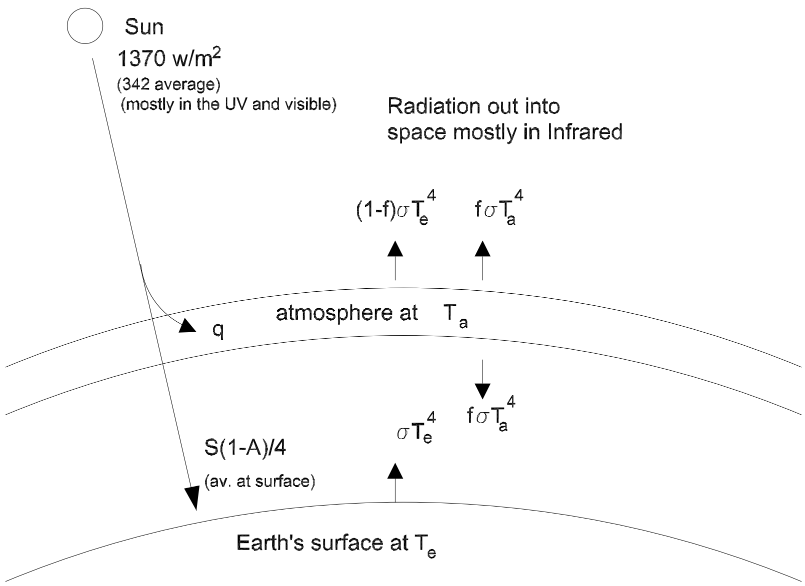

Figure 2. Energy balance for a layer in the atmosphere

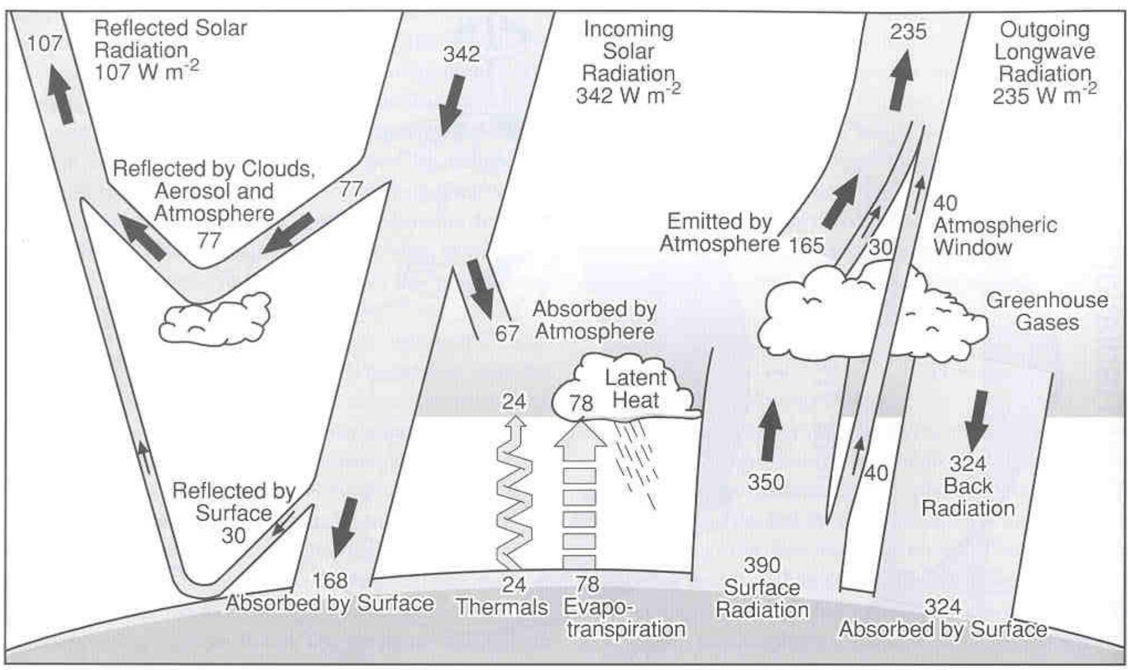

Figure 3. Illustrative energy budget for the Earth from "The Physics of Atmospheres," by John Houghton.