VSRT MEMO #069

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To: VSRT Group

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Subject: Simplified derivation of the conversion of observed ozone line temperature to ozone density concentration.

Introduction

This memo describes an approximate model for the observed 11 GHz ozone line as a means of explaining the model described in memo #33.

Simple model

In this model we assume the mesospheric volumetric density (ppmv) is all contained in a uniform thin layer of thickness d km and constant temperature T K located at an altitude of h km. Further we assume that the layer is observe with the antenna pointed at an elevation of θ radians.

1) Radiative transfer

   The temperature acquired by a ray passing through a thin layer of temperature T and opacity τ is approximately T τ.

2) Opacity

   The opacity of a spectral line in thermal equilibrium can be estimated from the integrated density given in the JPL tables.

   //spec.jpl.nasa.gov/cgi-bin/catform.html

   The value given for the 11 GHz ozone transition is -6.99.

   Since this is the value in logarithmic units of nm^{-2} MHz the opacity is:

$$10^{-6.99} \times 10^{-14} \times \rho \times L / w$$

   where ρ is the ozone density in molecules per cm^3, L is the path length in cm and w is the line width in MHz.

3) Ozone density in ppmv

   The ozone density in molecules per cm^3 is given by

$$P = n \times 10^{-6} \times P \times N_A \times W_{air} \times 273/(48 \times T)$$

   where n= number density in ppmv
   P = pressure in units of atmospheres

N_A = Avogadro's number (6.022 x 10^{23} $mole^{-1}$)
W_{air} = weight of 1 cubic cm of air at 1 atmosphere
48 = molecular weight of ozone
T = ozone temperature

4) Pressure at height h

$$P \sim 10^{-h/15.35} \text{ atmospheres}$$

5) Line width

The line width at the mesosphere is dominated by "Doppler Broadening" from the motion of the molecules. The width can be derived approximately by realizing that the kinetic energy of motion (1/2 mv^2) is equal to KT or more exactly

$$w = f_{line} \times \sqrt{(2 \ln(2) K_B T/m)}/c$$

where f_{line} ==11.072 x 10^3 MHz
K_B = Boltzmann's constant 1.38 x 10^{-23}
m = 48 x 1.67 x 10^{-27}
c= 3 x 10 ^{8} m/s

from which we get w = 7 kHz half power half width or 14 kHz half power full width.

6) path length

The path length is given approximately by:

$$\Delta h/\sin(\theta)$$

Where Δh is the layer thickness in altitude.

However at h=92 km the effect of the Earth's curvature is significant and needs to be taken into account as shown in figure 1 of memo #33.

7) Combining

If the terms are combined the line temperature is given by

$$T \times 10^{-6.99} \times 10^{-14} \times \rho \times (d/\sin\theta)/w$$

= 1 mK/ppmv with 1/sin θ

= 0.7 mK /ppmv accounting for the Earth's curvature

when T = 175 K
d = 5 km
w = 14 Hz
h = 92 km
θ = 8 degrees

The more accurate method using the shape of the spectrum results in a coefficient of 0.8 mK/ppmv at the line peak.