To:       Deuterium Array Group

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Subject:  Calculation of the Gray Chip filter response

The GrayChip GC4016 digital receiver chip performs the following calculations:

1. Multiplies 14-bit inputs by 20-bit digital sine and cosine functions
2. reduces the products to 20-bits 2s complements
3. passes the sine and cosine outputs into a Hogenauer filter (Hogenauer, IEEE trans on acoustics, ASSP-29, No. 2, April 1981 pp. 155-162) consisting of a CIC filter of 5 integrators a decimator by factor of R followed by 5 comb filters.
4. coarse gain scaler
5. a decimate by 2 low pass FIR filter - CFIR.
6. a decimate by 2 low pass filter - PFIR
7. 16 bit outputs for real and imaginary components.

The CIC filter is equivalent to 5 FIR filters in cascade each filter consisting of R coefficients of 1. The impulse response of this filter can be calculated by 4 successive convolutions of the R wide uniform filters. The impulse response of the overall GrayChip can then be obtained by convolving every 2nd point of the CIC impulse response with the impulse response of the 21 tap CFIR filter followed by a convolution of every 2nd point with the impulse response of the 63 tap PFIR filter. The overall “bandpass” for a white Gaussian input can be computed by averaging the spectrum of the impulse response over all possible time placements of an input impulse that will produce a non zero output in the time window of the FFT.

\[ h_i(t) = 1 \text{ for } \tau = 0, \ldots, R-1 \text{ and zero elsewhere} \]

\[ h_{CIC}(\tau) = h_0 \otimes h_1 \otimes h_2 \otimes h_3 \otimes h_4 \]

\[ h_{CFIR}(\tau) = 5, -8, -172 \ldots \text{(see cfr_80 of GC4016)} \]
\[ h_{C1C2}(\tau) = h_{C1C}(2\tau) \]
\[ h_{CFIR}(\tau) = h_{C1C2} \otimes h_{CFIR} \]
\[ h_{CFIR2}(\tau) = h_{CFIR}(2\tau) \]
\[ h_{PFIR}(\tau) = 31,136,\ldots \text{(see pfr\_80)} \]
\[ h_{GRAY}(\tau) = h_{CFIR2} \otimes h_{PFIR} \]
\[ x_\tau(t) = \mu_0(\tau) \otimes h_{\text{Gray}}(\tau) \]

where \( x(t) \) is the input to the FFT

\[ \text{bandpass } (w) = \left\langle |x_\tau(w)|^2 \right\rangle \]

The calculated response which is shown in figure 1 will be used to remove the filter ripple by dividing the spectrum by this response. This can be done in the final stages on the processing in the host computer.

![Figure 1. GC4016 response to white noise using cfr\_80 and pfr\_80](image)