MASSACHUSETTS INSTITUTE OF TECHNOLOGY
HAYSTACK OBSERVATORY

WESTFORD, MASSACHUSETTS 01886

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Telephone: 781-981-5407
Fax: 781-981-0590

To: Mark 5 Development Group

From: Alan E.E. Rogers

Subject: Dispersion and temperature effects in coax cables

While the frequency dependence of cable attenuation is well known other frequency dependent effects are usually not mentioned in data sheets. Of particular interest to geodesy is the phase stability. The general view is that the fractional change in electrical length will be independent of frequency but if the cable impedance differs from the nominal value multiple reflections on the cable can effect the phase delay.

A] Effects of standing waves on the cable

For an impedance of $50+\delta$ the reflection coefficient at each end of the cable is

$$\Gamma = \delta/(100+\delta)$$

So that the phase at the cable end is changed by $\phi = a|\Gamma|^2 \sin(2w\tau)$

Where τ = group delay of cable
a = cable attenuation (power ratio)
= 2-way voltage attenuation

For example if $\delta \sim 10$ and a changes by 20% in going from 25 to 125 C then ϕ can change by about 0.2 degrees which corresponds to a change of 55 ps. In addition the cable impedance may change with temperature leading to an additional corruption of the phase delay of

$$\Delta\phi = 2a\Gamma\Delta\Gamma \sin(2w\tau)$$

If the cable impedance changes by 1% and there is a 0.1 reflection coefficient the resulting phase change can be as large as 0.06 degrees or 16 ps at 10 MHz.

B] Dependence of cable phase delay temperature coefficient on dielectric, expansion and resistive loss coefficients.

The complex cable propagation constant, γ is related to the cable capacitance, inductance and loss by

$$\gamma^2 = \alpha^2 - \beta^2 + 2\alpha\beta = (R + jwL)(G + jwC)$$

where $\gamma = \alpha + i\beta$

R = resistance per unit length
L = inductance per unit length
G = conductance per unit length
C = capacitance per unit length

Solving for the phase term, *β,* in the case of small dielectric loss (i.e. G=0)

$$\beta \simeq w\sqrt{LC}\left(1 + R^2/\left(8w^2L^2\right)\right)$$

$$\alpha \approx \frac{R}{2}\sqrt{\frac{C}{L}}$$

where w = angular frequency

If the temperature changes there will be changes in *R,L* and *C* as well as a physical expansion of the cable so that

$$\Delta\varepsilon = K_\varepsilon \varepsilon$$

$$\Delta R = 1/2\, K_R R$$

$$\Delta\ell = K_\ell$$

where $K_\varepsilon, K_R, K_\ell$ are the coefficients for the changes in dielectric, resistance and physical length respectively. Substituting these quantities the overall fractional coefficient for the cable is approximately:

$$\tfrac{1}{2}K_\varepsilon + K_\ell + K_R\left(R^2/\left(8w^2L^2\right)\right)$$

For LMR-240 cable at 10 MHz

$$wL \sim 0.1\Omega/cm$$

$$R \sim 3x10^{-3}\,\Omega/cm$$

So that the coefficient for the resistive loss term is

$$112K_R ppm$$

but since K_R is only 0.004/degC for copper the net coefficient from the change in resistance is only about +0.5 ppm.

In addition to the change in propagation constant with changes in resistance the "internal" inductance of the center conductor and shield changes with resistance. The theory is quite complex and nicely formulated in Ramo and Whinnery, Wiley 1953. While an accurate estimate of the change in inductance requires the use of Bessel functions BER, BEI and their derivatives the theory can be approximated in the case of small skin depth by internal inductance equal to *R/w*. In this case the fractional change in cable is approximately

$$\frac{1}{4}\left(\frac{R}{wL}\right)K_R$$

$$= (3/4)\frac{V_f dB_{100} K_R}{2\pi f \log_{10} e} \times 10^{11}\, ppm$$

where Vf = cable velocity factor

dB100 = cable loss at frequency f in dB/100m

$$= \frac{110 V_f \; dB_{100}}{fMHz}\, ppm/^{\circ}C$$

$$\approx 24 \text{ ppm/deg C}$$

at 10 MHz which is close to the trend seen in the measurements.