Super-resolution Polarimetric Imaging of Black Holes using the Event Horizon Telescope

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Abstract
Black holes are thought to reside in the centers of many galaxies; however, due to their diminutive size, we have yet to directly detect and image a black hole. The Event Horizon Telescope (EHT), a global array for 1.3mm very long baseline interferometry (VLBI), has been designed to observe and image the supermassive black hole in the center of the Milky Way (Sagittarius A*), as well as the one in the center of the nearby giant elliptical galaxy M87. The nominal resolution of the EHT is around 30 μas, comparable to the size of the black hole’s event horizon. For this reason, we require super-resolution to accurately reconstruct images in total intensity and linear polarization. High fidelity polarimetric imaging can be used to test general relativity and to characterize the magnetic field structure surrounding black holes, which is important for understanding its role in mediating the innermost accretion and outflow region. We employ new sparse imaging techniques based on compressed sensing for linear polarimetry. Using synthetic data of M87 observations with the EHT, we find that our new techniques improve upon the standard CLEAN by a factor of ten regardless of resolution, as measured by the differences in mean squared error (MSE). We conclude that compressed sensing proves to be an effective method for linear polarimetric imaging.

Motivation
- Black holes are deduced to play a role in the formation and evolution of galaxies, so accurate black hole models can help to better our understanding of galactic evolution
- Polarimetric imaging can help us understand the structure of the black hole jet’s magnetic field and its relation to the emitting gas
  - Directly imaging a black hole using the EHT will provide crucial information to improve simulations to model jet formation and propagation
- Other EHT objectives include testing general relativity, understanding jet generation and collimation in galaxies, and characterizing accretion around black holes

The Event Horizon Telescope

Methods
We employ a method of sparse modelling based on compressed sensing for the first time to reconstruct super-resolution polarimetric images of black holes. This method is an imaging technique that works in the visibility domain. It is based on the idea of sparsity, i.e. that the majority of pixels in the image have a value of zero. We determine which reconstruction best represents the true image by imparting regularizers. We mathematically encode prior knowledge as to what the image should look like and utilize minimization techniques to find the best image. We impart Cross Validation (CV) (a statistical method used to assess how well a model will generalize to an independent data set), to help determine the best image created by sparse modelling. It helps us to determine the weight of the regularizers that yield the highest fidelity images. We compare our results to the traditional CLEAN method to determine which method yields the highest fidelity images.

Regularizers
Our compressed sensing technique imparts regularization factors to reach a solution. We must determine what type of regularization factor and how much of this factor to impose on the data.

- L1-norm: Favors a sparse solution
- TV-norm: (Total Variation): Favors a smoother solution

\[ \min_{i} \frac{1}{2} \| V \|_2^2 + \lambda_1 \| i \|_1 + \lambda_2 \| TV(\| i \|) \]

In function, these two regularization parameters can yield the highest fidelity image, provided the weights for each factor are optimized.

Images
For each model, the top image depicts total intensity, while the bottom image depicts the P-map. The dashed lines indicate polarization angle and their color indicates the fractional polarization. From left to right we compare the truth image, to the image convolved at 15 μas, to the CLEAN reconstruction, Pure-L1, Pure-TV, L1&TV regularized to Q,U separately, and L1&TV regularized to Pmap. We notice that CLEAN and Pure-L1 reconstructions for total intensity are similar to each other (as expected: CLEAN implicitly using an L1 regularization parameter). In the P-map images, Pure-TV fails to accurately reconstruct the images. For the total intensity images, using L1+TV with compressed sensing produces better images than CLEAN. We therefore conclude that L1&TV regularizers are required for our compressed sensing technique.

Image Fidelity Results
For each model, we analyze the goodness of our image reconstruction for each of the Stokes parameters using the Mean Square Error (MSE) for CLEAN, Pure-L1, Pure-TV, L1&TV reconstructions for (Q,U,P)-map. We compare CLEAN to compressed sensing with beam sizes of 5 μas, 15 μas, and 25 μas to determine the best reconstruction for super-resolution regimes.

Conclusions
- We find that sparse modelling can reconstruct higher fidelity images than CLEAN by a factor of 10
  - At finer beam sizes, sparse modelling produces better images
- For polarimetric imaging, it is essential to use L1 and TV regularizers to obtain the highest fidelity image
  - Pure-L1 or Pure-TV proves to be insufficient when applied alone
- Whether we regularize P-map or (Q,U)-maps separately, our results are similar to within a few percent