Analysis of and Improvements upon Covariance Estimation Algorithms for Direction of Arrival Estimation using Vector Sensors

Daniel Dopp
University of Kentucky

Project Mentor: Ryan Volz
Project Goals

1. Develop a software package ready for integration with researchers scientific workflow - ✓
2. Characterize the strong points and limitations of covariance estimation as a DoA estimation approach - ✓
3. Support other REU projects and collect data in the field - ❌
4. Investigate ways to improve algorithm performance – in progress
Background: Vector Sensors

- Compact sensor sensitive to all directions and polarizations of EM waves.
- Provides complete description of magnetic field at a single point in space.
- Allows for more science in less space.
- Allows for direction finding using a single antenna element.
Background: DoA Estimation

• How do we go from a sample signal to recovering the direction of arrival?

• Assume our sample is a combination of contributions from different parts of the sphere

• We can make a guess at the direction of arrival of the signal sources, and see how likely it is that DoA generated the signal we read.
Background: Proximal Gradient Descent

Vector Sensor → Source Signal → Measure Sample Covariance → DoA Guess → Proximal Gradient Descent Algorithm → Source Estimate

Final DoA Estimate

Yes! Estimate Good Enough? Yes! No, update & try again
Example: Successful Convergence
Example: Successful Convergence

Algorithm convergence for 2 points, Stokes I Parameter

Iteration: 0
Example: Slowdown and local minima
Example: Slowdown and local minima

Algorithm convergence for 3 points, Stokes I Parameter
Example: Great circle degeneracy
Example: Great circle degeneracy

Algorithm convergence for 3 points, Stokes I Parameter
Algorithm Performance: Accuracy

Distance weighted squared error averaged over 20 simulations per parameter set

Distance weighted squared error

Number of signal sources

Angular area (square degrees)

Distance weighted squared error
## Algorithm Performance: Speed

Mean evaluation time over 20 simulations per parameter set

<table>
<thead>
<tr>
<th>Angular area (square degrees)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.84</td>
<td>76.77</td>
<td>324.21</td>
<td>1632.70</td>
<td>802.92</td>
<td>404.75</td>
<td>210.93</td>
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<tr>
<td>3.36</td>
<td>18.81</td>
<td>34.34</td>
<td>497.05</td>
<td>218.38</td>
<td>92.88</td>
<td>100.02</td>
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<td>13.43</td>
<td>4.26</td>
<td>11.80</td>
<td>279.54</td>
<td>231.72</td>
<td>73.76</td>
<td>32.76</td>
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<tr>
<td>53.71</td>
<td>0.61</td>
<td>1.15</td>
<td>43.11</td>
<td>24.20</td>
<td>21.12</td>
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<tr>
<td>214.86</td>
<td>0.64</td>
<td>1.24</td>
<td>34.97</td>
<td>22.07</td>
<td>12.80</td>
<td>7.44</td>
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</tbody>
</table>

Number of signal sources
Algorithm Improvements: L1 Regularizer
Algorithm Improvements: L1 Regularizer

Algorithm convergence for 3 points, Stokes I Parameter

Iteration: 0

-0.9999 1.0001
Alternative Algorithms: Neural Networks

• Predictive model fit by training on example data.
• Moves computational burden from runtime to train time.
• Issues arose due to data starvation and small number of features
• Initial results show these methods likely aren’t well suited to general direction of arrival estimation problems in their current form.
Algorithm Performance: NN Methods

Distance weighted squared error averaged over 20 simulations per parameter set

<table>
<thead>
<tr>
<th>Number of signal sources</th>
<th>Neural Net</th>
<th>ProxGrad</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.34</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>3.70</td>
<td>0.00</td>
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<tr>
<td>3</td>
<td>5.03</td>
<td>1.56</td>
</tr>
<tr>
<td>4</td>
<td>5.48</td>
<td>1.60</td>
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</table>

Distance weighted squared error
Takeaways

• Covariance estimation algorithms are highly effective at point source DoA estimation for small source numbers

• Accuracy and computation time suffer as source numbers and angular area restrictions are increased

• Regularization techniques provide an avenue to resolve higher source numbers

• Alternative estimation techniques may provide an avenue to resolve higher source numbers or cut down on computational cost
Future Works

• Integrate software into existing Haystack radio toolkits
• Move tests from simulation to the real world
• Further investigate regularization techniques to improve the performance of proximal gradient descent
• Further investigate the uses of alternative optimization algorithms
Acknowledgments

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• Tim Morin
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