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## To: EDGES group

From: Alan E.E. Rogers
Subject: Simulations of the use of scaled antennas for global 21-cm measurements
Global 21-cm measurements using scaled antennas has been suggested and studied by Bang Nhan and Richard Bradley of NRAO. This project is known as "Scaled Antennas For Ascertaining the Radio Index (SAFARI)".

In this memo the use of scaled antennas to reduce the effect of beam chromaticity is simulated using FEKO.

Simulations are made using an EDGES-3 antenna and a planar dipole with 73.4 cm long by 95.3 cm wide 88 cm above the ground plane at 75 MHz . A frequency range of 55 to 110 MHz was used and 5 physical terms used for the foreground. A square ground plane of $4 x 4 \mathrm{~m}$ at 75 MHz was chosen for the scaled antennas and FEKO scaled the dipole and the dipole with its ground plane were re-scaled every 2 MHz . The data was simulated by convolving the beam with the Haslam map scaled to $50-110 \mathrm{MHz}$. The EDGES 2018 absorption centered at 78 MHz with depth of 0.5 K , width of 19 MHz and tau $=7$ flattening parameter was added to the map. The simulated data was processed using the same receiver calibration but without beam correction. The results for a grid search for the best fit using 5 physical terms and a fixed value of tau $=7$ are shown below in Table 1 .

| Center | SNR | sig | width | rmsin | rms | ant | ght | scaled site | comments |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 78.1 | 78 | 0.47 | 19.8 | 63 | 8 | dipole | 0 | Y | WA | Soil 1e-2 6 -18 GHA |
| 78.1 | 66 | 0.58 | 18.7 | 87 | 14 | dipole | 0 | Y | WA | freespace |
| 78.1 | 110 | 0.49 | 19.7 | 66 | 6 | dipole | 0 | Y | WA | rock |
| 78.1 | 96 | 0.42 | 19.2 | 60 | 7 | dipole | 0 | N | WA |  |
| 70.7 | 33 | 7.6 | 30 | 718 | 223 | dipole | 1 m | N | WA | Very large systematics |
| 77.4 | 21 | 0.57 | 22.8 | 58 | 27 | dipole | 1 m | Y | WA | ght also scaled |
| 77.4 | 21 | 0.57 | 22.9 | 59 | 27 | dipole | 1 m | Y | WA | ght fixed at 1m |
| 77.7 | 62 | 0.46 | 18.6 | 69 | 12 | dipole | 1 m | Y | WA | ght fixed at 1m 0-24 GHA |
| 77.7 | 56 | 0.45 | 18.7 | 68 | 13 | dipole | 1 m | Y | WA | ght also scaled 0-24 GHA |
| 78.1 | 130 | 0.54 | 18.8 | 80 | 7 | EDGES-3 | N | WA | $0-24$ GHA |  |
| 78.1 | 118 | 0.47 | 19.1 | 67 | 6 | EDGES-3 | N | Wake | $0-24$ GHA |  |
| 78.9 | 30 | 3.58 | 16.5 | 703 | 235 | dipole | 0 | ant Y | WA | $0-18$ GHA gnd not scaled |

Table 1. Simulations of scaled dipole plus ground plane and EDGES-3 on $48 x 48 \mathrm{~m}$ ground plane
The units of the center frequency and width are MHz. rmsin and rms are the residuals with 5-physical terms removed in mK before and after fitting the absorption. ght is the height of the ground plane above the ground when elevated.

These results show the following:
1] The use of scaled antennas does "cancel" the effects of the beam chromaticity so that is it not necessary to make any beam corrections.

2] Based on the last entry it is necessary to scale both the antenna and the ground plane showing that the ground plane is indeed a critical part of the antenna.

3] Based on the entries for the antenna with ground plane being elevated above the ground by 1 m the cancellation of the of the beam effects are degraded even if the height above the ground is scaled.

4] The performance of a single EDGES-3 with the large 48x48m ground plane antenna looks to be comparable with what can be achieved with scaled antennas.

Tests are now made of the calibration as the simulations in table 1 which assume a perfect calibration. In addition to measuring the S11 of the scaled antennas and making corrections it would be possible to use the method of "Galaxy Calibration" to correct the calibrations of each scaled antenna. The results of Galaxy calibration using the method described in memo 247 using the difference spectrum

$$
\begin{aligned}
D=\left(G_{d}-r G_{u}\right) /(1-r) \text { where } \quad G_{d} & =\text { spectrum centered at GHA }=12 \text { hours } \\
G_{u} & =\text { spectrum centered at GHA }=0 \text { hours } \\
\mathrm{r} & =\text { ratio } G_{d} / G_{u} \\
\mathrm{D} & =\text { difference corrected on the assumption that the absorption is } \\
& \text { global }
\end{aligned}
$$

The columns in table 2 are the same as in table 1 . The simulated spectrum has been adjusted by changing the antenna S11 adb, the LNA S11 ldb and the antenna loss.

| center | SNR | sig | width | rmsin | rms | ant | site | comments |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 77.7 | 60 | 0.61 | 19.9 | 54 | 9 | dipole | WA | adb 0.2 dB |
| 78.1 | 235 | 0.48 | 18.8 | 45 | 2 | dipole | WA | ldb 2 dB |
| 77.7 | 118 | 0.53 | 19.4 | 47 | 4 | dipole | WA | loss $5 \%$ |
| 78.1 | 117 | 0.48 | 19.1 | 44 | 4 | EDGES-3WA | adb 0.2 dB |  |
| 78.1 | 51 | 0.59 | 18.4 | 58 | 11 | EDGES-3WA | ldb 2 dB |  |
| 77.7 | 75 | 0.45 | 19.3 | 40 | 5 | EDGES-3WA | loss 5\% |  |
| 77.7 | 53 | 0.64 | 20.1 | 56 | 6 | dipole Wake | adb 0.2 dB |  |
| 77.7 | 43 | 0.42 | 19.6 | 38 | 9 | EDGES-3Wake | adb 0.2 dB |  |

Table 2. Galaxy calibration simulation results
These simulations show that it should be possible to correct the calibration of the individual spectrometers on each antenna by using the observations of the Galaxy up and Galaxy down data from each day. The Galaxy_down/up ratios are 0.33 for the WA (formerly named MRO) and 0.48 for Wake Island respectively.

These simulations are based on broad beam antenna pointed at the zenith. In this case they should be placed in a line about 20 meters apart to minimize the effects of cross coupling between antennas.

The next test is to simulate an array of scaled antennas on corner reflectors proposed by Bang Nhan and Richard Bradley. In this case the Kraus 1938 design is used with a center fed half wave rod dipole a half wave above the corner. The actual size at 75 MHz chosen for the first test used a 4 m long corner with 2 m high dipole and 2 m high edges. The length of the dipole is adjusted to get the best match of 8.28 dB . The antenna gain at the zenith is 9.68 dBi and the gain at the horizon ranged from -23 to -22 dBi with soil diel 3.5 and $1 \mathrm{e}-2 \mathrm{~S} / \mathrm{m}$. The loss due to the ground was estimated to be about $2 \%$. Doubling the size of the reflector increases the zenith gain to 12.45 dB and lowers the horizon gain to a range of -31 to -23 dBi and the ground loss is essentially zero.

| Center | SNR | sig | width | rmsin | rms | ant | spc | scaled site | comments |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 78.1 | 155 | 0.49 | 19.1 | 45 | 2 | corner | 2 | Y | WA | no loss |
| 78.1 | 140 | 0.59 | 19.2 | 46 | 3 | corner | 2 | Y | WA | $2 \%$ loss |
| 77.7 | 65 | 0.57 | 19.6 | 51 | 8 | corner | 2 | Y | WA | adb 0.2 dB |
| 78.1 | 84 | 0.53 | 19.2 | 49 | 6 | corner2 | 2 | Y | WA | adb 0.2 dB |
| 78.1 | 109 | 0.47 | 19.2 | 43 | 4 | corner | 2 | Y | WA | ldb 2 |
|  |  |  |  |  |  |  |  |  |  |  |
| 89.9 | 15 | 0.37 | 18.8 | 35 | 20 | EDGES-3 |  | WA | adb 0.2 Gal. Cal nobeam |  |
| 77.7 | 87 | 0.46 | 19.2 | 41 | 5 | EDGES-3 |  | WA | adb 0.2 Gal. Cal with beam |  |

Table 4. Sensitivity to systematics for a spacing of 2 MHz .
These simulations show that the corner reflector antennas are less sensitive to errors in antenna S11 than EDGES primarily because there is less frequency structure in the antenna S11. In addition antenna S11 errors and receiver calibration errors can be corrected using the Galaxy calibration without an accurate knowledge of the beam whereas Galaxy calibration of EDGES is useful as a check it needs knowledge of the beam for an accurate Galaxy calibration. Corner2 in table 4 is for a corner reflector twice the size. The EDGES-3 single antenna simulation is shown in table 4 as a comparison. The cases of added systematics errors in loss, antenna S11 and LNA S11 are listed in the comments.

The next test is to find number of antennas needed for an array to cover $50-100 \mathrm{MHz}$ because a 2 MHz spacing requires 26 antennas and receivers. Table 5

| Center | SNR | sig | width | rmsin | rms | spacing MHz site | comments |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 78.1 | 187 | 0.49 | 18.9 | 45 | 2 | 2 | WA | GHA 6-18 |
| 78.1 | 191 | 0.49 | 18.8 | 46 | 2 | 5 | WA | GHA 6-18 |
| 78.1 | 188 | 0.50 | 18.8 | 47 | 2 | 10 | WA | GHA 6-18 |
| 77.7 | 45 | 0.42 | 19.3 | 38 | 8 | 10 | WA | GHA 6-18 adb 0.5 |
| 78.1 | 136 | 0.5 | 18.6 | 47 | 4 | 10 | WA | GHA 0 12 adb 0.5 Galcal |
| 78.1 | 167 | 0.49 | 18.7 | 46 | 3 | 10 | WA | GHA 0\&12 adb 0.5 ldb 2 |
| 78.1 | 101 | 0.52 | 18.5 | 49 | 5 | 10 | WA | Same as above only 6 antennas |
| 78.1 | 44 | 0.12 | 17.9 | 13 | 0 | 10 | WA | See text absorption 0.10 |

Table 5. Tests of the spacing of antennas with added systematic errors listed in comments
Tests with simulation covering 50 to 110 MHz with final absorption search covering 55 to 100 MHz yield very good results with antenna spacing of 10 MHz which would require 7 antennas or 6 antennas
with to cover 50 to 100 MHz with a little degradation of performance. The solutions break down with only 5 antennas spaced 12 MHz apart. The last entry is a simulation for which the absorption added to the sky is only 0.1 K deep but bear in mind while systematic errors listed in the comments column have been added that no noise has been added to these simulations. The simulations in Table 5 use a reflector of 2 x 4 m at 75 MHz . The results may be a little better with larger reflectors but this will make a larger array.

The next test is to find the separation needed between the antennas to avoid significant coupling. In order to make the compute time reasonable FEKO is used to derive the beam of each antenna with the scaled antenna on each side. For the 10 MHz spacing there is only one other antenna next to the antenna whose beam is being modeled.

| Center | SNR | sig | width | rmsin | rms | ant | len m | comments |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 78.1 | 188 | 0.50 | 18.8 | 46 | 2 | corner |  | GHA 6-18 no coupling |
| 78.1 | 93 | 0.55 | 18.8 | 51 | 5 | corner | 66.14 | GHA 6-18 with coupling |
| 78.1 | 183 | 0.47 | 19.0 | 44 | 3 | corner2 | 132.2 | GHA 6-18 with coupling |
| 78.5 | 31 | 0.69 | 18.1 | 74 | 23 | corner | 66.14 | GHA 0\&12 adb 0.5 ldb 2 |
| 78.1 | 45.2 | 0.62 | 18.7 | 59 | 13 | corner2 | 132.3 | GHA0\&12 2adb 0.5 ldb 2 |
| 78.1 | 86 | 0.53 | 18.4 | 59 | 6 | corner | 66.14 | GHA0\&12 2adb 0.5 ldb 2 freespace |

Table 6. Tests of cross-coupling for frequency spacing 10 MHz 7 antennas $50-110 \mathrm{MHz}$
Two cases are simulated. In the first case of the seven reflectors are a half wavelength high and one wavelength long and in the second case the reflector size is doubled. In the first case the adjacent edges of the reflectors are separated by 1.5 wavelengths and by 3 wavelengths in the second case. The total length is 66.14 and 132.3 meters for the 2 cases respectively. The antennas are placed so the corners are on the ground along a straight line. All the FEKO simulations in tables 2 to 6 are on soil with a dielectric of 3.5 and conductivity of $1 \mathrm{e}-2 \mathrm{~S} / \mathrm{m}$. The last entry is for the reflector array in free space which shows that the scaled corner reflector array is probably not very sensitive to the type of soil below.

Figure 1 shows a sample of a FEKO model used in the simulations of the reflector array.
The choice of the larger array is clearly better as the correction of antenna S11 and receiver calibrations is marginal with the half wavelength high reflectors. The best choice may lie between the two choices of size.

The next test is to simulate data from the second case corner reflector array with random independent errors from each of the antennas. For this case I chose to only use data from 6 antennas 50 to 100 MHz . I used the FEKO beams at each 10 MHz convolved with the Haslam map and antenna S11 for each antenna from FEKO. I added an independent random error of 3 gaussian ohms to the real and imaginary complex impedance derived from the FEKO S11 of each antenna. The antenna S11 values were then fit with a 5-term polynomial shown in the top left plot of Figure 2. The simulated data was then processed with a typical EDGES-3 calibration using the FEKO S11 data with a different set of random errors and fitted with a 4-term polynomial shown in the top left plot in Figure 2. The simulation using the same random errors used to simulate the effects of differences in the electronics and its calibrations was run for 1 hour blocks of GHA from 6 to 18 hours at the MRO. The whole simulated data set was then processed and the results of the grid search for the Nature feature, which was added to the sky map, is shown in the lower left plot of Figure 2 along the residues for 5-physical terms removed is shown in the lower left plot. There was no beam correction made in the processing.

For this simulation 2 MHz of bandwidth of data for each spectrometer is combined into a 50 to 100 MHz band by fitting with a 30 term Fourier series with zero weight at frequencies not within more than a MHz from a 2 MHz band of each spectrometer.

The overall conclusion is that provided the errors in calibration and reproducibility are adequately accounted for by the S11 errors added in the simulations the scaled antenna array should perform as well as or better than the EDGES. While the EDGES system gives a reasonable result without beam correction as reported in Bowman et al. 2018 the scaled antenna array is insensitive errors in the sky model as the sky model is not used in the estimation of the $21-\mathrm{cm}$ absorption. The only potential use of a sky model is for removing spectrometer calibration errors.

In summary, based of these simulations, the scheme of using scaled reflectors looks very good and is probably better than the broadband single antenna approach of the EDGES systems at least if enough support is available for the larger amount of hardware and electronics.

$x^{7}$

| Altair FekO"' | test7 | View direction <br> Theta $=62^{\circ}$ <br> Phi $=32^{\circ}$ |
| :---: | :---: | :---: |

Figure 1. Sample FEKO model used to simulate the corner reflector array.





Figure 2. Simulations of a scaled antenna array using added errors in antenna s11 to test the dependence of differences between the spectrometers.

