Estimation of the instantaneous Earth rotation

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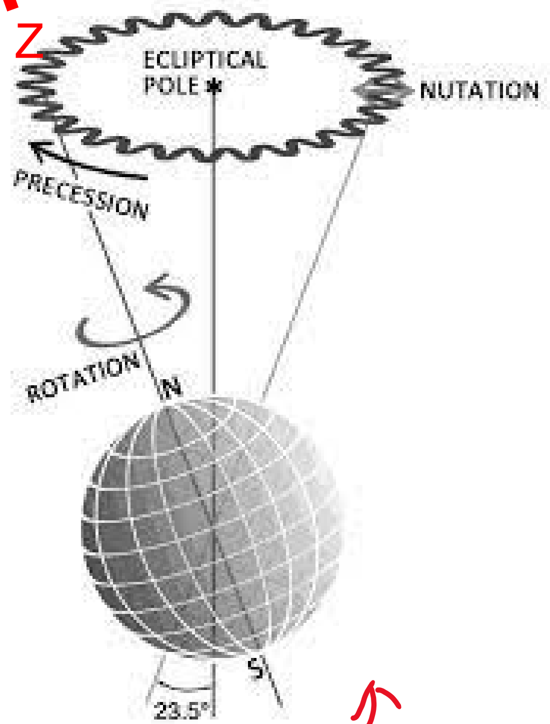
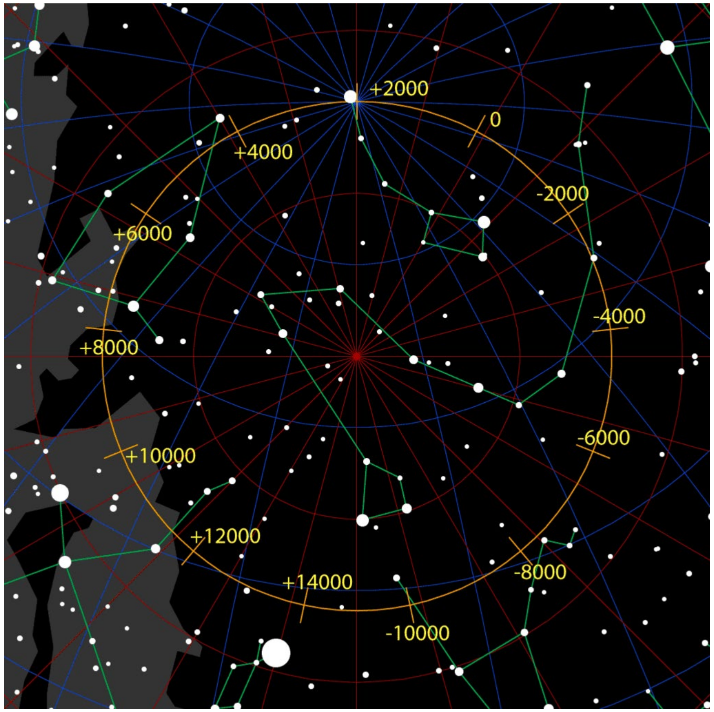
Precession and nutation

X, Y, Z



$\Omega \approx 7.2 \cdot 10^{-5}\ \ 1/s$



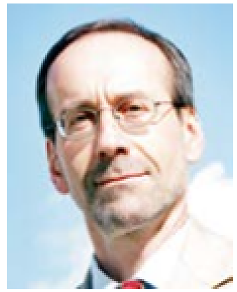
Precession: period of approximately 26,000 years or 1° every 72 years (50”/year)

Nutation: period of 18.6 years, amplitude ~ 17” in longitude, ~9” in obliquity

Vermessung & Geoinformation 2/2007, P. 113 – 120, 8 Figs.

The instantaneous Earth rotation – still inaccessible?





Paulo Jorge Mendes Cerveira, Robert Weber and Harald Schuh

Abstract

Nowadays, space geodesy, such as Very Long Baseline Interferometry (VLBI) and Global Navigation Satellite System (GNSS) and Satellite Laser Ranging (SLR), allows determining Earth orientation to a fraction of 1 milliarcsecond with daily to hourly resolution. This should not prevent us from studying other innovative and powerful technologies. A new emerging technology, called ring laser gyroscope, is a high-precision tool that provides us with extra information in the daily and sub-daily time domain. The experimental determination of the amplitudes of the forced diurnal polar motion is for example exclusively allocated to the ring laser technique. Another aspect, in which ring lasers could emphasize their supremacy, is the determination of the motion of the Earth-fixed frame w.r.t. the instantaneous Earth rotation axis. However, present ring lasers are huge constructions extremely sensitive to external effects, e.g., temperature variations.

This paper illuminates the relationship between various Earth rotation axes, in the large sense, and discusses the separation between polar motion and nutation of these axes in space and w.r.t. the Earth body. The second part covers the expected benefit of ring laser observables.

L. Petrov, A&A, 2007, 467, 359

The fundamental problem is that no observation technique, except the laser gyroscope, is sensitive to the instantaneous Earth's rotation vector or to its time derivatives *directly*. The rotation angles can be derived using the least square estimation procedure, together with evaluating other parameters. It requires accumulating sufficient amount of data in order to separate variables. The estimates of the Earth's rotation angles cannot be sampled too fast. A typical sampling rate of estimates is one day, since this allows compensation for a certain type of systematic error. In some cases the sampling rate can be reduced to several hours.

Article

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Variations in the Earth's rotation rate measured with a ring laser interferometer

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Check for updates

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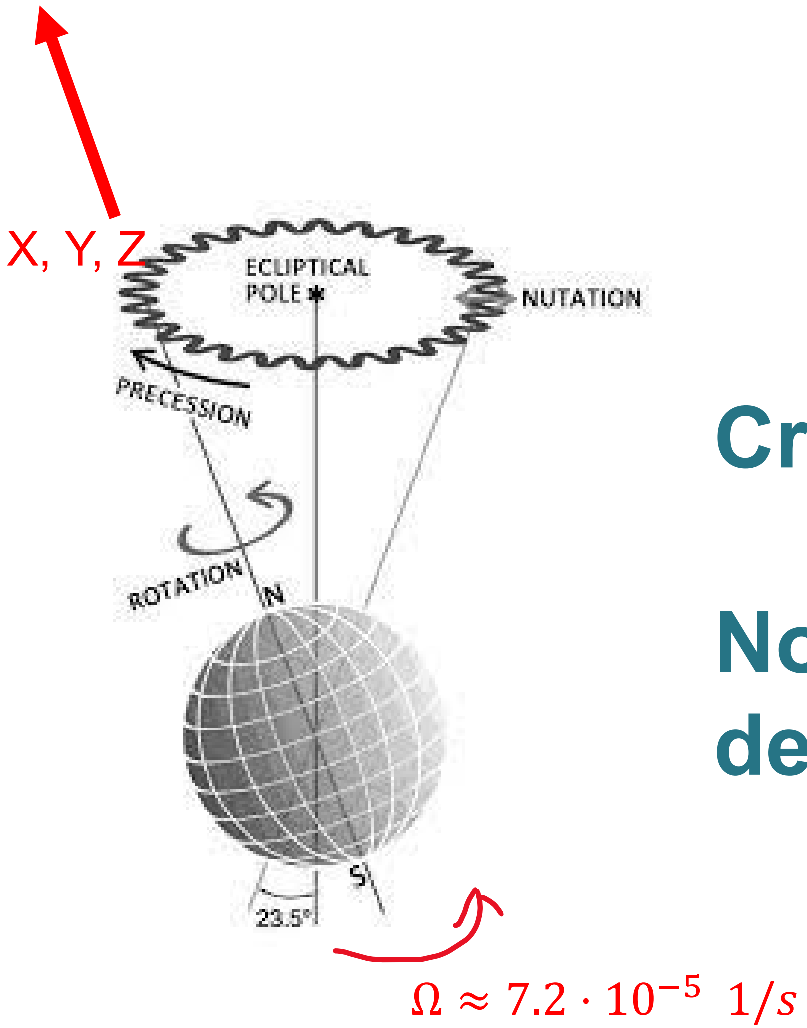
An exact knowledge of the instantaneous Earth's rotation rate is indispensable for accurate navigation and geolocation. Fluctuations in the length of sidereal day are caused by momentum exchange between the fluids of the Earth (namely, the atmosphere, hydrosphere and cryosphere) and the solid Earth. Since a multitude of different globally distributed and independent mass transport phenomena are involved, the resultant effect on the Earth's rotation is not predictable and needs to be continuously measured. Here we report the observation of minute variations in the rotation rate of the Earth at the level of five parts per billion, namely, with a resolution of a few milliseconds over 120 days of continuous measurements. We employ an inertial self-contained measurement technique based on an optical ring laser interferometer rigidly strapped down to the Earth's crust and operated in the Sagnac configuration. This large-scale gyroscope integrates over three hours for each data point, as opposed to an entire global network of Global Navigation Satellite Systems receivers and Very Long Baseline Interferometry that can only provide a single measurement per day.



Critical parameters (X, Y, Ω)

No accessible with the VLBI delay only

VLBI measures delay and delay rate (relative shift of interference frequency)

$$\tau = -\frac{(b_{21} \cdot s)}{c}.$$

Used

$$\frac{\Delta f_{21}}{f} = -\frac{([\Omega \times b_{21}] \cdot s)}{c} = -\frac{([s \times b_{21}] \cdot \Omega)}{c}$$

Never used

Ω is the accessible with delay rate!

Being estimated the Ω could get the accuracy of $\frac{\Delta\Omega}{\Omega} = 10^{-8} \div 10^{-9}$

VLBI measures delay and delay rate (relative shift of interference frequency)

$$\frac{\Delta f_{21}}{f} = -\frac{([\Omega \times b_{21}] \cdot s)}{c} = -\frac{([s \times b_{21}] \cdot \Omega)}{c}$$

observations

partials

parameters

Estimating of Ω

$$\frac{\Delta f}{f} \approx -\frac{(w_2 - w_1, s)}{c} = -\frac{(\Omega \times b, s)}{c} = -\frac{(b \times s, \Omega)}{c}$$

$$\frac{\Delta f}{f} \approx \boxed{\frac{1}{c}\begin{pmatrix} 0 & -b_1 s_3 & b_1 s_2 \\ b_2 s_3 & 0 & -b_2 s_1 \\ -b_3 s_2 & b_3 s_1 & 0 \end{pmatrix}}\begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{pmatrix}$$

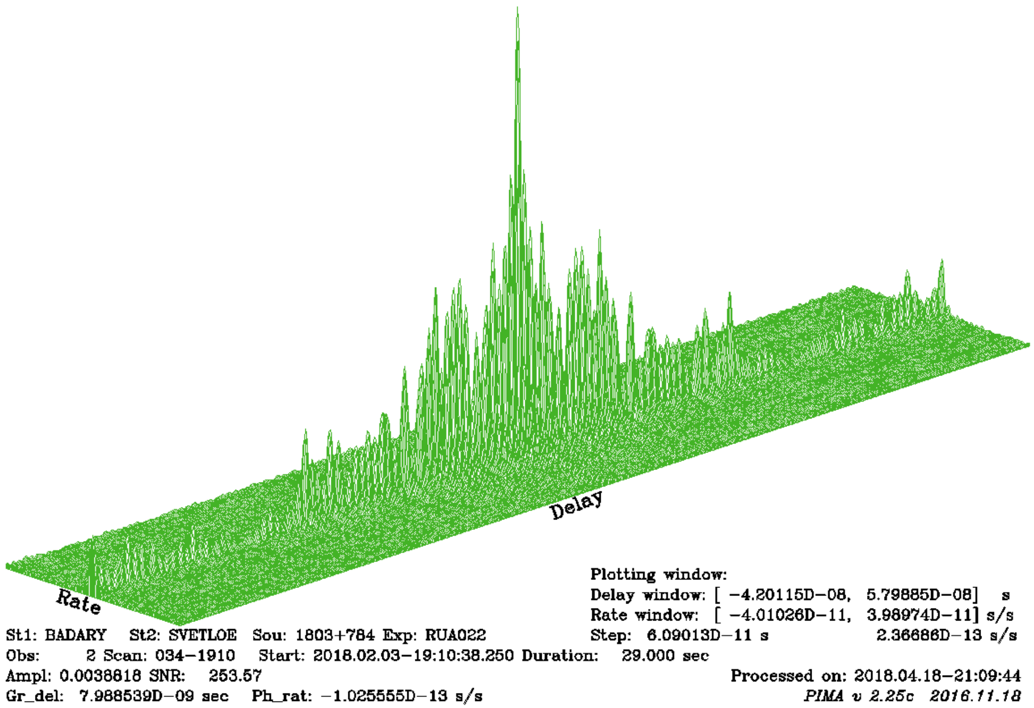
$$\frac{\Delta f}{f} \simeq 1\ \mu\text{rad}$$

$$\sigma_{\frac{\Delta f}{f}} \simeq 10^{-14} \div 10^{-15}\ \text{rad}$$

Three components of the Earth angular rotation vector

Fringe is two-dimensional

Two measurable values– delay and delay rate



$$\tau \approx -\frac{(r_2 - r_1, s)}{c} = -\frac{(b, s)}{c}$$

$$\frac{\Delta f}{f} \approx -\frac{(\Omega \times b, s)}{c}$$

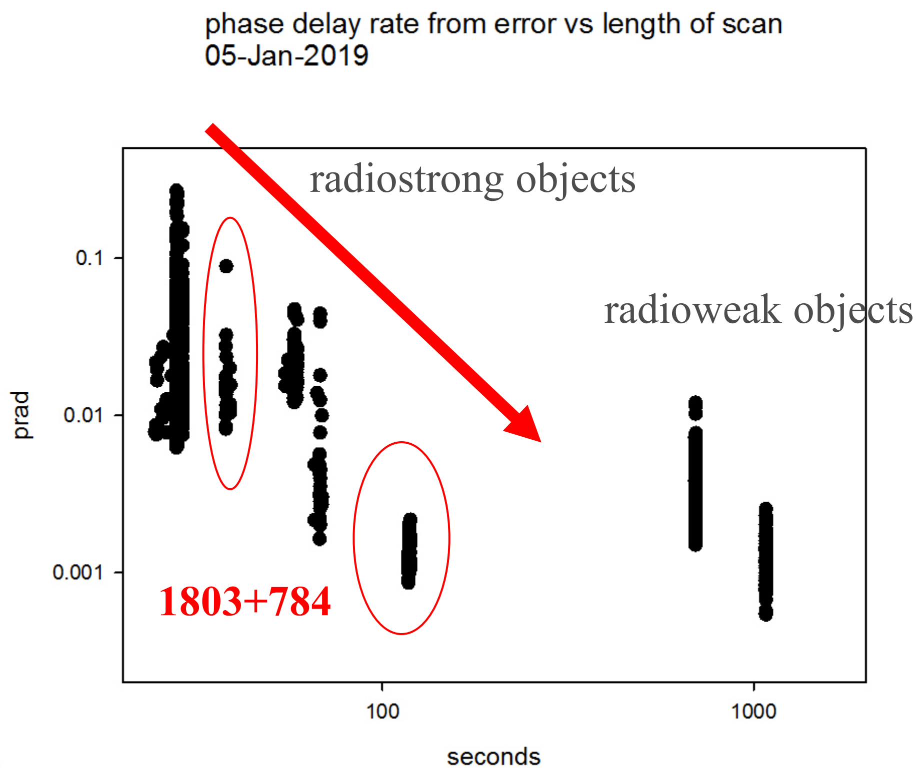
b = 6000 km,

$$\frac{\Delta f}{f} \simeq 1\ \mu\text{rad}$$

$$\sigma_{\frac{\Delta f}{f}} \simeq 10^{-14} \div 10^{-15}\ \text{rad}$$

Delay rate formal error

Formal errors of delay rate depends on many factors, i.e. integration time, SNR, etc



$$\sigma_{\frac{\Delta f}{f}} \simeq 10^{-14} \div 10^{-15} \text{ rad}$$

formal error vs integration time $\sim \Delta t^{-3/2}$

Quasar 1803+784, integration time is 30 sec and 120 sec

$$\Omega = 7.292115 \cdot 10^{-5} \text{ 1/sec}$$

Delay rate is better than group delay because…

- No clock phase instability
- No clock phase breaks
- Troposphere wet delay is less critical
- Radio source positions and station coordinates are less critical
- ALL modelling deficiencies are less critical
- Plain least squares method with a limited number of parameters is good enough

VLBI data analysis

- OCCAM 6.3
- NEOS-A and IVS-R1, R4 experiments from 1993 (~4500 exps)
- IERS Conventions model (precession-nutation, solid tides, etc)
- Relativistic models for geodetic VLBI delay rate (special and general relativity effects)
- Plain least squares method
- Estimated parameters: clock (3 per station), wet troposphere delay and delay rates with gradients (6 per station), and Ω (3 components)

VLBI data analysis – relativistic model

$$\frac{\Delta f_{21}}{f} = -\frac{((w_2 - w_1)\cdot s)}{c}\left(1 - \frac{2GM}{c^2 r} + \frac{(V_\oplus \cdot s)^2}{c^2} - \frac{\langle V_\oplus{}^2\rangle}{2c^2} - \frac{(V_\oplus \cdot s)}{c} - \frac{(w_2 \cdot s)}{c} - \frac{(V_\oplus \cdot w_2)}{c}\right) +$$
$$+ \frac{(b_{21}\cdot s)}{c}\frac{(a_\oplus \cdot s)}{c} + \frac{(b_{21}\cdot s)}{c}\frac{(a_2 \cdot s)}{c} - \frac{(b_{21}\cdot a_\oplus)}{c^2} +$$
$$+ \frac{\left((w_2 - w_1)\cdot V_\oplus\right)(V_\oplus \cdot s)}{2c^3} - \frac{((w_2 - w_1)\cdot V_\oplus)}{c^2}$$

w_1 and w_2 are vectors of geocentric velocities;

r is the geocentric distance to the Solar centre;

s is the vector of direction to the radio source,

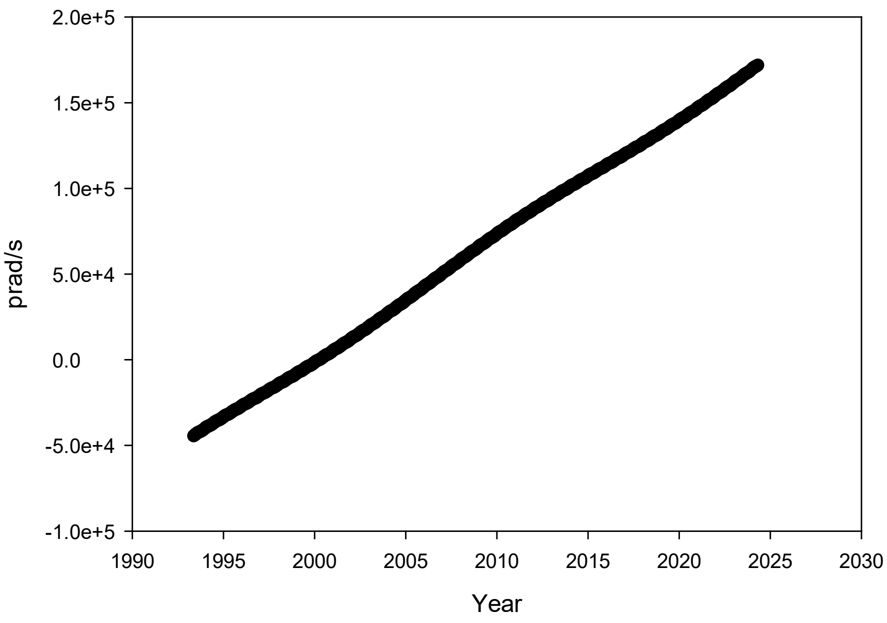
b_{21} is the baseline vector, c is the speed of light, $V_\oplus$ is the barycentric velocity of the Earth;

$a_\oplus$ is the barycentric acceleration of the Earth, a_2 is the geocentric acceleration of the second station

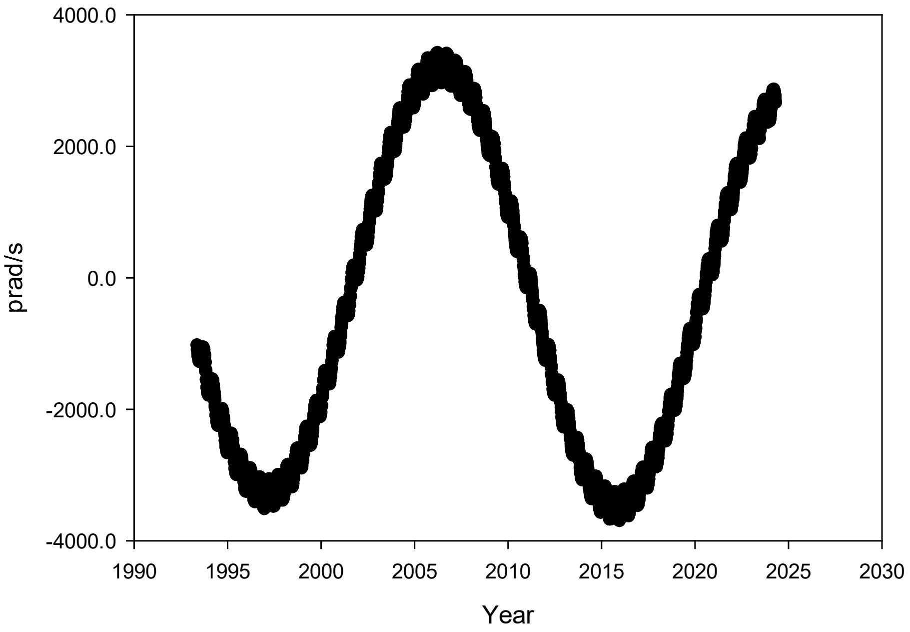
$$\frac{\Delta f_{21}}{f} = -\frac{2GM}{c^3}\frac{(\Omega \cdot [b_{21} \times [s \times [s \times N]]])}{(|r_2| - (s \cdot r_2))}$$

Vector Ω component estimates (~4500)

ω_x



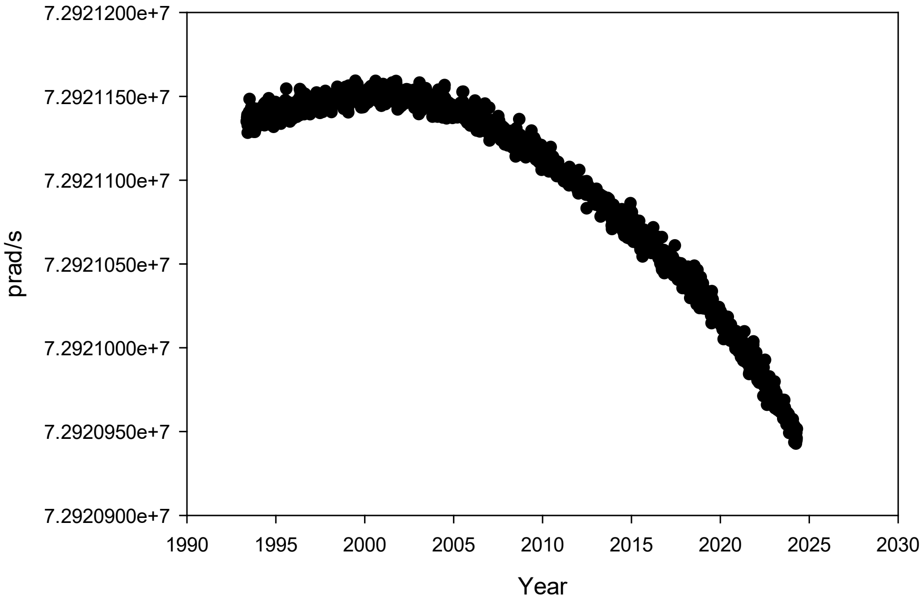
ω_y



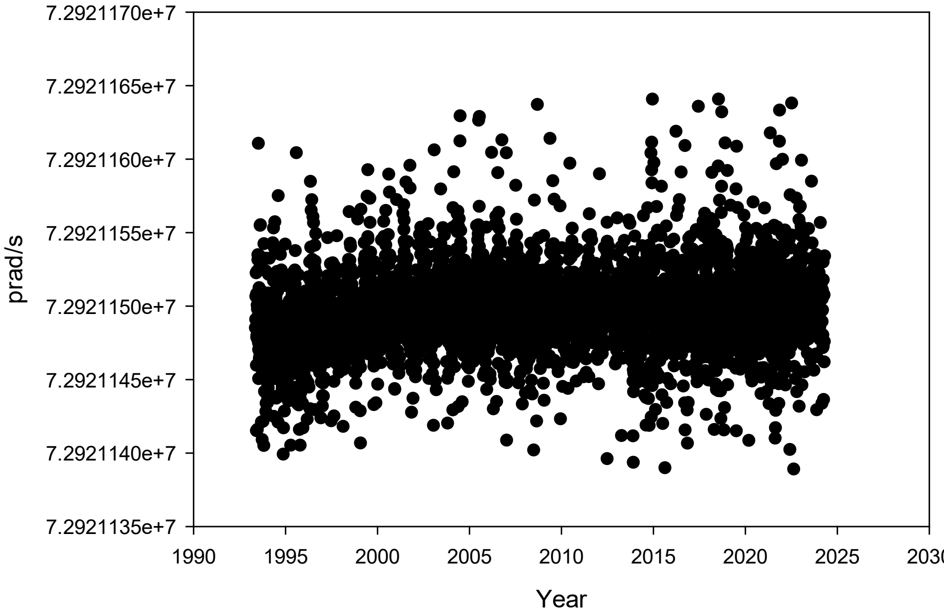
Relative formal errors are about 10^{-8}

Vector Ω component estimates (~4500)

ω_z



Ω magnitude



Instantaneous angular rotation vector

Interval is 24 hours, but the observation time could be shortened

Transformation to X and Y

$$\omega_{\mathrm{x}} = \sin X$$
$$\omega_{\mathrm{y}} = \cos X \sin Y$$
$$\omega_{\mathrm{z}} = \cos X \cos Y$$

$$X = \operatorname{arctg} \frac{\omega_x}{\sqrt{\omega_y^2 + \omega_z^2}}$$
$$Y = \operatorname{arctg} \frac{\omega_{\mathrm{y}}}{\omega_z}$$

Precession - Nutation model

$$X(t) = \sum_{i=0}^{3} a_i \Delta t^i + \sum_{i=1}^{7} \left(A_i^c \cos\frac{2\pi\Delta t}{P_i} + A_i^s \cos\frac{2\pi\Delta t}{P_i} \right)$$

$$Y(t) = \sum_{i=0}^{3} b_i \Delta t^i + \sum_{i=1}^{7} \left(B_i^c \cos\frac{2\pi\Delta t}{P_i} + B_i^s \cos\frac{2\pi\Delta t}{P_i} \right)$$

i	a_i (")	b_i (")
0	-0.04354 ± 0.00081	0.00515 ± 0.00072
1	20.04104 ± 0.00016	0.00092 ± 0.00016
2	-0.000057 ± 0.000030	-0.002428 ± 0.000028
3	-0.0000031 ± 0.0000011	0.0000056 ± 0.0000010

p·sinε ≈ 20".04, (p – precession constant, 50"/year; ε – ecliptic inclination, 23°.5)

Nutation model (7 parameters)

$$X(t) = \sum_{i=0}^{3} a_i \Delta t^i + \sum_{i=1}^{7} \left(A_i^c cos \frac{2\pi\Delta t}{P_i} + A_i^s cos \frac{2\pi\Delta t}{P_i} \right)$$

$$Y(t) = \sum_{i=0}^{3} b_i \Delta t^i + \sum_{i=1}^{7} \left(B_i^c cos \frac{2\pi\Delta t}{P_i} + B_i^s cos \frac{2\pi\Delta t}{P_i} \right)$$

Period (year)	A_i^c (“)	A_i^s (“)	B_i^c (“)	B_i^s (“)
0.037402 (13.6 d)	-0.0749 ± 0.0004	-0.0314 ± 0.0004	0.0356 ± 0.0004	-0.0816 ± 0.0004
0.075443 (27.555 d)	0.0206 ± 0.0004	-0.0176 ± 0.0004	0.0007 ± 0.0004	-0.0018 ± 0.0004
0.3333	0.0059 ± 0.0004	0.0165 ± 0.0004	-0.0203 ± 0.0004	0.0071 ± 0.0004
0.5	0.1792 ± 0.0004	0.4872 ± 0.0004	-0.5359 ± 0.0004	0.1965 ± 0.0004
1.0	-0.0065 ± 0.0004	0.0460 ± 0.0004	0.0151 ± 0.0004	-0.0044 ± 0.0004
9.3	-0.0773 ± 0.0005	0.0281 ± 0.0004	0.0296 ± 0.0004	0.0846 ± 0.0004
18.6	-5.6069 ± 0.0005	-3.9266 ± 0.0008	-5.2931 ± 0.0005	7.5288 ± 0.0008

Angular rotation vs Length of Day

$$\Omega = \left(\omega_x^2 + \omega_y^2 + \omega_z^2\right)^{1/2} \approx (\mathrm{X}^2 + \mathrm{Y}^2 + \mathrm{Z}^2)^{1/2}$$

By now we had the LOD only. Now we could compare LOD and Ω from VLBI

$$\mathrm{LOD} \approx -\frac{\Omega - \Omega_0}{\Omega_0}$$

Standard approximation

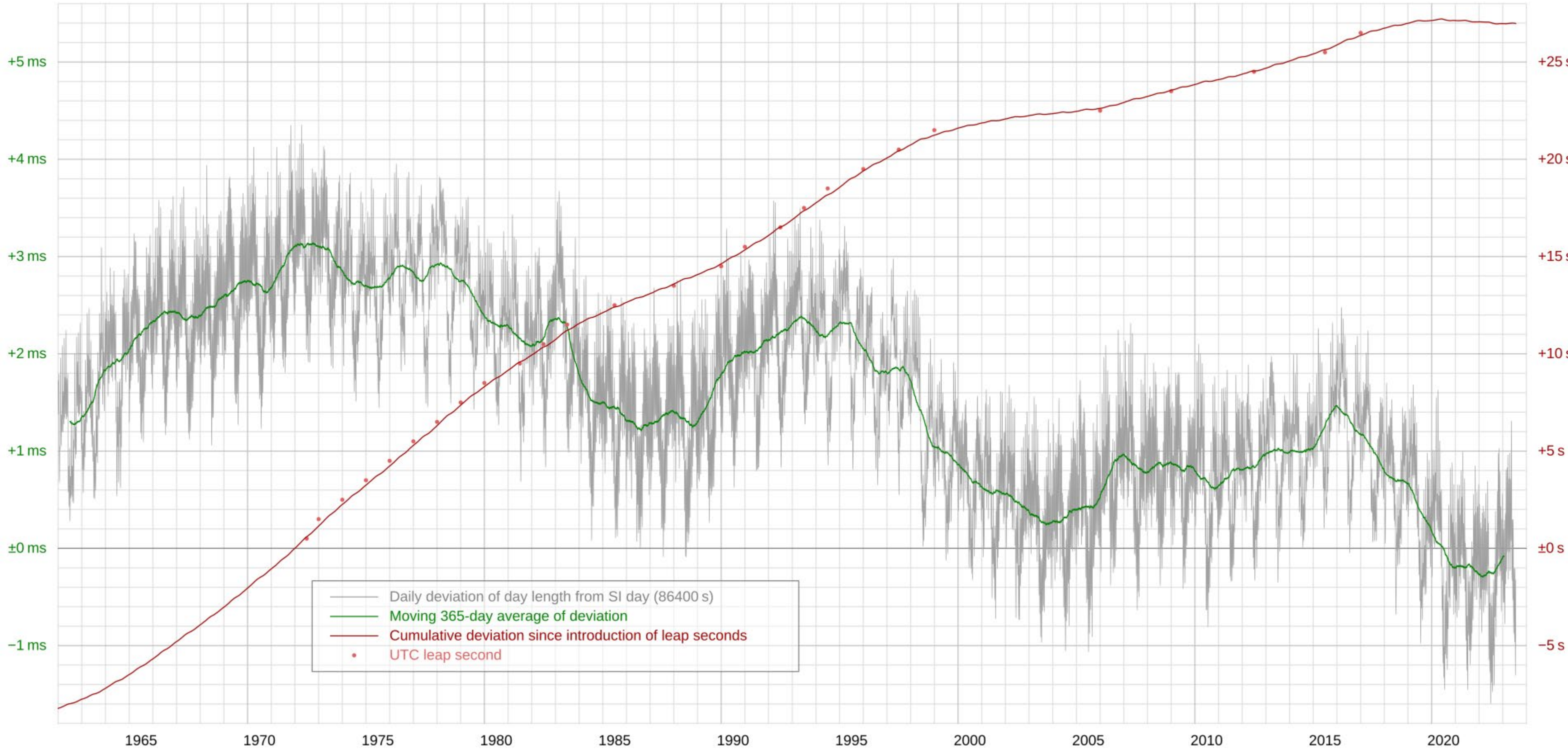
$$\frac{\Delta\mathrm{LOD}}{\mathrm{LOD}} \approx -m_3$$

LOD is a small increment to Ω
The axial moment of the Earth inertia (m_3) is blamed for all variations of the LOD

$$\frac{\Delta\mathrm{LOD}}{\mathrm{LOD}} \approx -m_3 - X\dot{Y} + Y\dot{X}$$

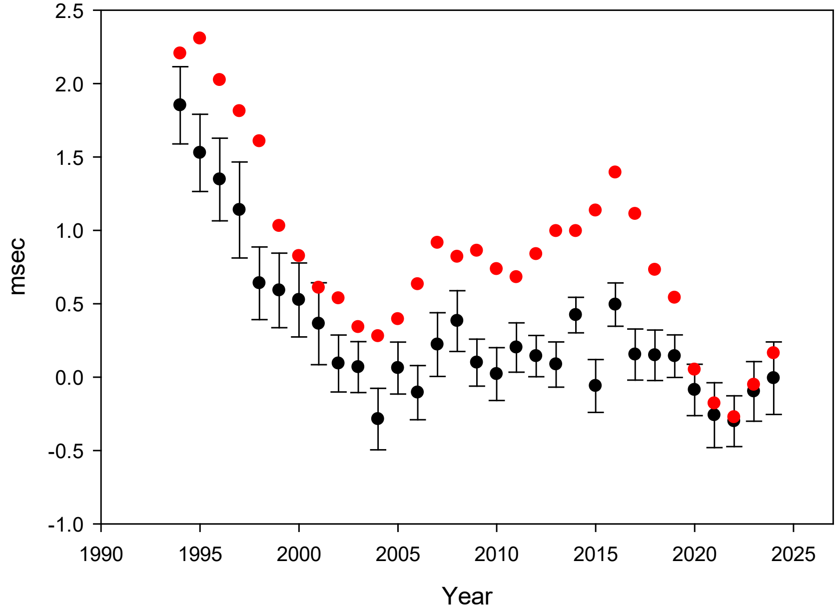
If Ω is measured independently in a full scale, one could extract contribution of the nutation (18.6-year)

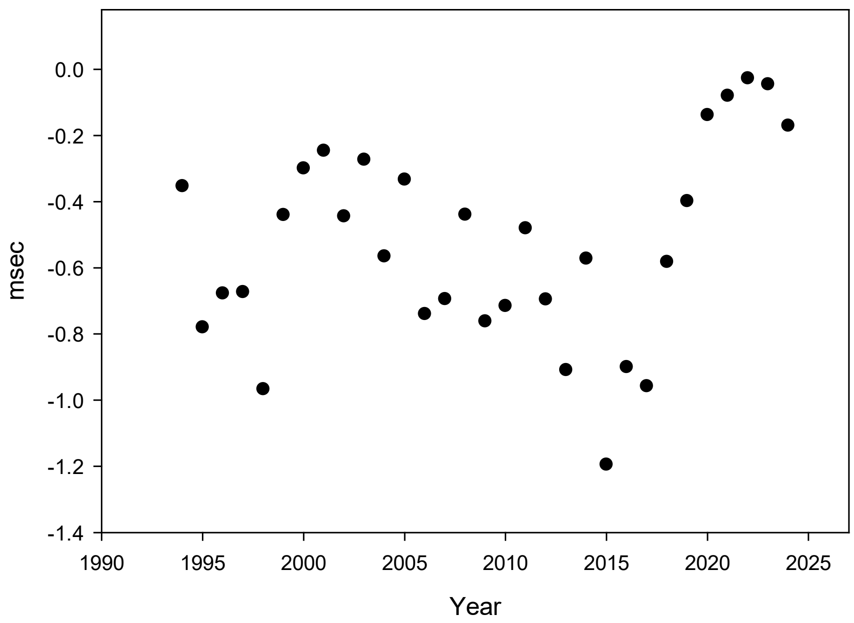
LOD (IERS)



(18.6-year signal in LOD)

LOD vs LOD (IERS vs Ω-based)



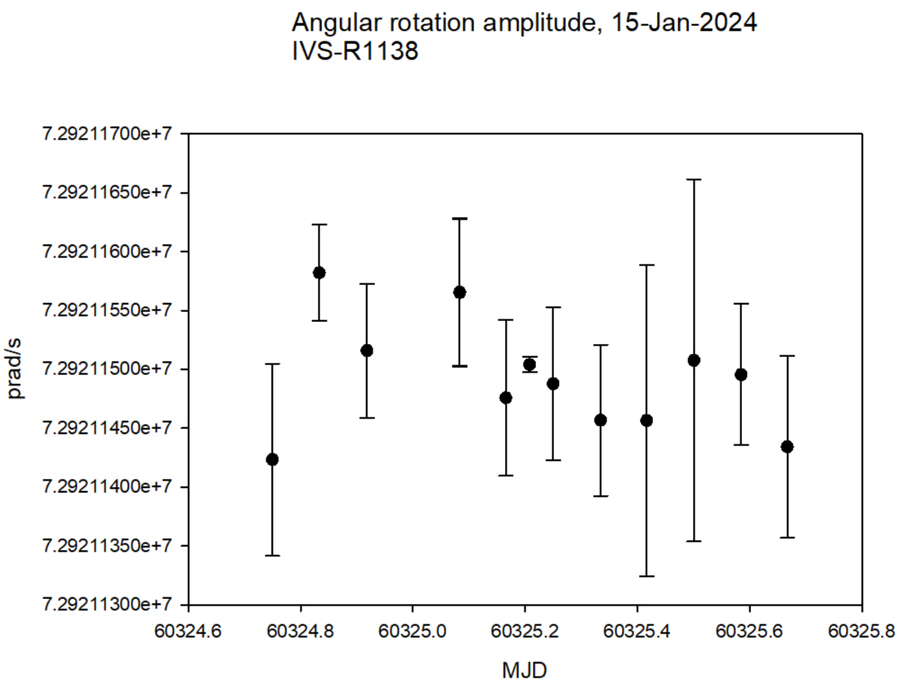


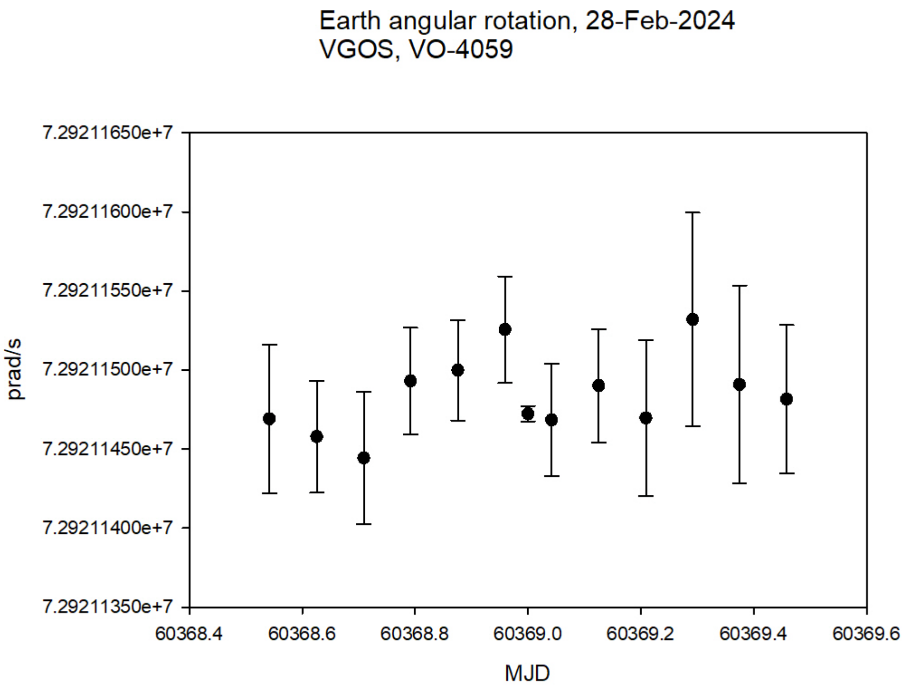
Difference between two LODs, probably caused by the nutation effect

$$\mathrm{LOD} \approx -\frac{\Omega - \Omega_0}{\Omega_0}$$

$$-X\dot{Y} + Y\dot{X}$$

Time resolution situation





Time resolution is 2 hours, but ring laser interferometer announced 3 hours

Time resolution improvement

1. Dedicated network/schedule
2. 24/7 mode
3. Near-online correlation
4. Kalman filter technique

1 estimate each 1-2 minutes with formal error of 10^{-8} or better

Conclusion

1. *Nutation model could be tested directly with the X, Y times series*

2. *LOD is not a full equivalence of the Ω. Variations of LOD could be caused by the X, Y terms rather than axial component perturbations*

3. *Time resolution is 2 hour is shown; a dedicated network may provide 1-hour resolution or better*

$$\frac{\Delta \mathrm{LOD}}{\mathrm{LOD}} \approx -m_3 - X\dot{Y} + Y\dot{X}$$

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