RFI MEMO #030

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To: RFI Group

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Subject: Correction antenna temperature for cable attenuation and antenna mismatch

A difficult problem in radiometry is the correction for the effects of the antenna VSWR and cable between the Dicke switch and the antenna. In a 3 position switched radiometer

$P_A = g\left(T + T_R\right)$ - - power on antenna

$P_L = g\left(T_L + T_R\right)$- - Power on load

$P_c + g\left(T_L + T_c + T_R\right)$- power on load + calibration

From which

$$T = T_c\left[\left(P_A - P_L\right)/\left(P_c - P_L\right)\right] + T_L$$

where g = gain

P_A = power on antenna

P_L = power on load

P_c = power on load + cal

T_R = receiver system noise

T_c = excess calibration noise

T_L = ambient load temperature

If the antenna is connected via cable then the relation between the antenna temperature and the measured temperature can be modeled using the sum of the multiple reflections between the antenna and the LNA. The 2-way cable delay is

$$\tau = 2\ell/(vc)$$

where ℓ = cable length

v = velocity factor

c = free space velocity

The cable attenuation, *cab*, is approximately given by

$$cab = 3.0 \times \left(f/150\right)^{1/2} \times \ell/100\ dB$$

for LMR-240 whose one way loss is about 3 dB per 100 feet at 150 MHz. This loss results in a temperature T' reaching the LNA where

$$T' = T_A \left(1 - \left|\Gamma_A\right|^2\right) cab + T_L \left(1 - cab\right)$$

where Γ_A = antenna reflection coefficient

this ignores multiple reflections between the antenna and the LNA. If the LNA is mismatched

$$T_{obs} = T'\left(1 + 2a + a^2 + b^2\right) + T_{out}\left(c^2 + d^2\right) + \left(T_R T_{out}\right)^{1/2} 2c\rho + T_L\left(1 - cab\right)\left(c^2 + d^2\right) \Big/ cab$$

where $a = \sum_{j=1}^{\infty} \alpha^j \cos\left(2\pi f \tau j\right)$

$$b = \sum_{j=1}^{\infty} \alpha^j \sin\left(2\pi f \tau j\right)$$

$$c = a + \cos\left(2\pi f \tau\right) \left|cab \times \Gamma_A \times \left(1 - \left|\Gamma_a\right|\right)\right|$$

$$d = b + \sin\left(2\pi f \tau\right) \left|cab \times \Gamma_A \times \left(1 - \left|\Gamma_a\right|\right)\right|$$

$$\alpha = \left|cab \times \Gamma_A \times \Gamma_a\right|$$

Γ_a = LNA reflection coefficient

T_{out} = noise transmitted out of LNA input

ρ = correlation coefficient between output noise and noise transmitted out of the LNA input.

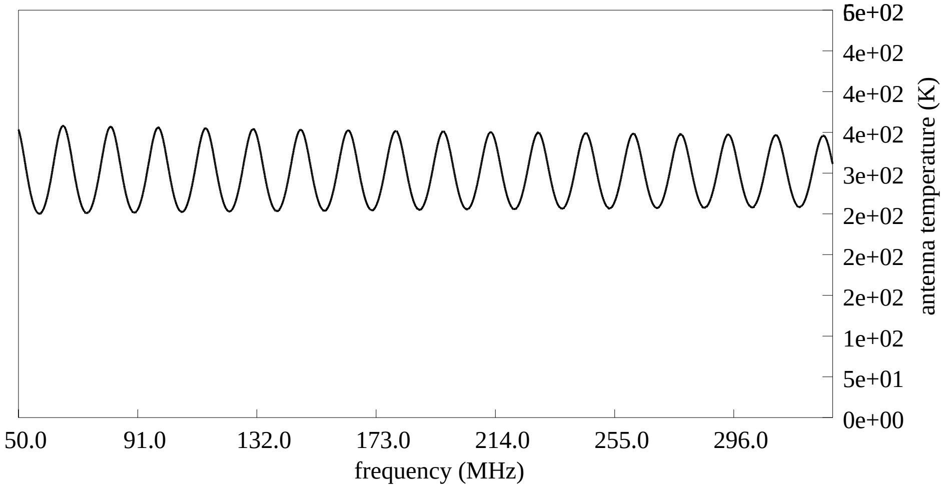
T_R and T_{out} are both approximately 20 K and the LNA input reflection coefficient is approximated by $0.98 \times 10^{-0.01(f/100)}$ while $\rho \sim 0.5$.

The parameters of this model which are difficult to separately measure with test equipment $\left(T_{out}, \rho \text{ and } \Gamma_a\right)$ can be checked by comparing the observed temperature over a wide frequency range with a long open ended cable is connected to the radiometer. Another check can be made by terminating the cable with a load of known reflection coefficient (for example at 10 dB pad given $\Gamma_A = 0.1$). With known parameters the amplitude of the observed periodic ripple can be used to measure the antenna reflection coefficient and correction of the observed temperature, averaged over one cycle of ripple, for the $\left(1 - \left|\Gamma_A\right|^2\right)$ loss factor. This modeling makes it possible to obtain the true corrected antenna temperature over a wide range of frequency without the complication of using a matching circuit which is an intrinsically narrow band device and has to be "tuned" for each observed frequency.

The 2-fold objectives of EDGES:

1] To measure the cosmic background magnitude and spectral index from 100 to 200 MHz with < 5% absolute accuracy.

2] To set limits on the smoothness of the variation of spectral index from 100-200 MHz and to apply this limits to model of the global EOR signature. The first objective may be best accomplished using a 25 foot (or longer) section of calibrated cable between the Dicke switched LNA and the antenna while the second objective will be best served with the shortest possible connection between the LNA and the antenna.



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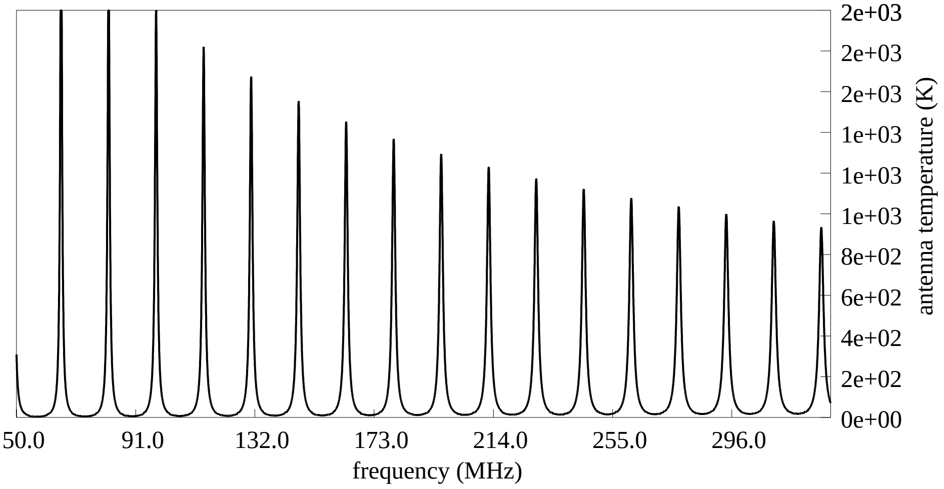
Mon Jul 24 17:43:38 2006

Fig 1. Modeled effect of $\Gamma_A = 0.1$ on $25'$ cable



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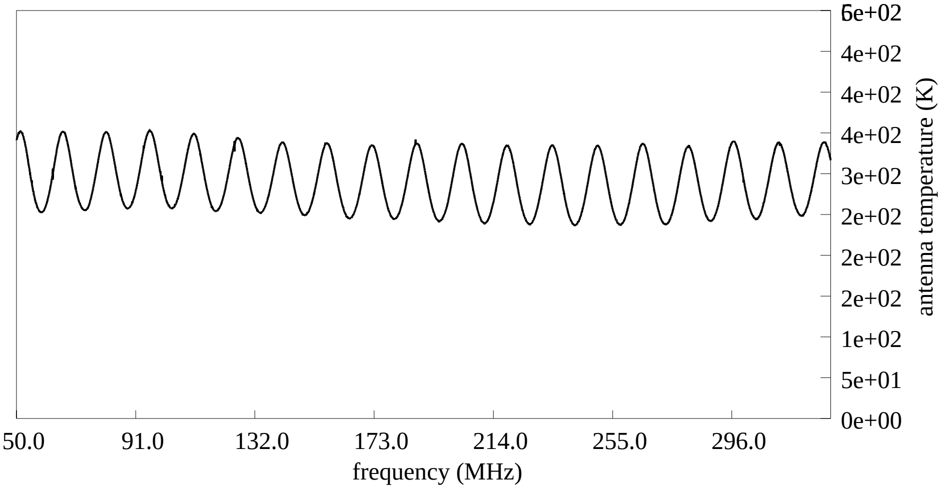
Mon Jul 24 16:54:20 2006

Fig 2. Modeled effect of open $25'$ cable



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Tue Jul 25 08:58:17 2006

Fig 3. Measured spectrum with $\Gamma_A = 0.1$ on end of $25'$ LMR-240

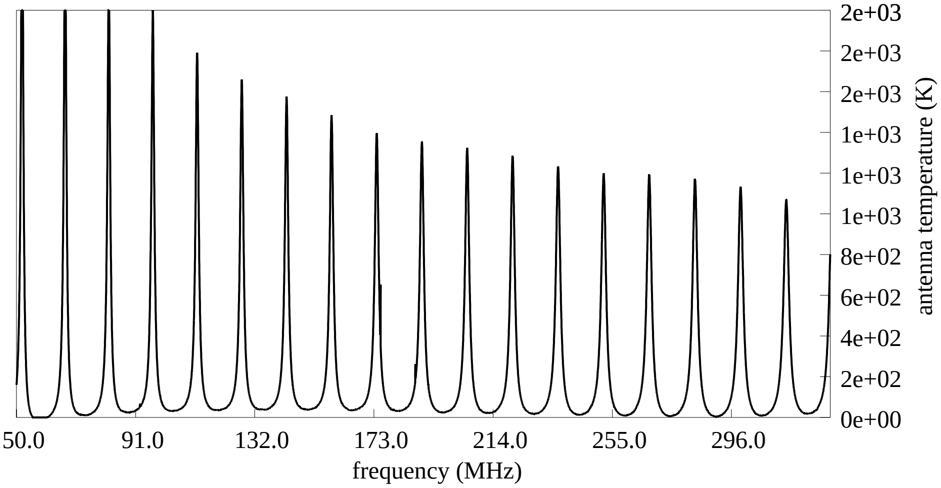

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Fig 4. Measured spectrum with 25′ LMR-240 open at the end.