

Coin Flipping Activity

Pascal's Triangle: The Historic Context*

In 1653, a French mathematician-philosopher Blaise Pascal described a triangular arrangement of numbers corresponding to the probabilities involved in flipping coins, or the number of ways to choose ' n ' objects from a group of ' m ' indistinguishable objects. Pascal's triangle has many uses in binomial expansions. Although Pascal never claimed recognition for his discovery, his name is inseparably linked with it. In fact, the triangle had been described centuries earlier. The first reference occurs in Indian mathematician Pingala's book on Sanskrit poetics, which may be as early as 450 BC, as Meru-prastaara, the "staircase of Mount Meru." Chinese mathematicians also knew about the triangle that was called "Yang Hui's triangle." Later, the Persian mathematician Karaji and the Persian astronomer-poet Omar Khayyám referred to the triangle as the "Khayyám triangle." Several theorems related to the triangle were known, including the binomial theorem. Finally, in Italy, it is referred to as "Triangolo di Tartaglia" (*Tartaglia's triangle*), named for the Italian algebraist Niccolò Fontana Tartaglia who lived a century before Pascal. Tartaglia is credited with the general formula for solving cubic polynomials.

The first seven rows of Pascal's Triangle look like:

1	r = 0
1 1	r = 1
1 2 1	r = 2
1 3 3 1	r = 3
1 4 6 4 1	r = 4
1 5 10 10 5 1	r = 5
1 6 15 20 15 6 1	r = 6
Note that every number in the interior of the triangle is the sum of the two numbers directly above it.	

*From: <http://www.archimedes-lab.org/pascaltriangle.html>

We recommend the following sites and applets for further investigation on the topic and to supplement your knowledge. One of the RET's spent many enjoyable hours at Boston University one summer working with fractals, nature's way of "flipping coins." There is a plethora of information and activities for students on probability; many of them are pleasurable.

Probability: http://www.mathgoodies.com/lessons/toc_vol6.html

http://math.bu.edu/DYSYS/applets/	http://argento.bu.edu/ogaf/html/chp3.htm
http://polymer.bu.edu/java/	http://argento.bu.edu/ogaf/
http://polymer.bu.edu/pins/index.html (Highly recommended)	

Introduction: Coin flipping is based on probability. With an honest coin, the chances of winning or losing are 50% and consequently, coin flipping is used to decide such momentous events like who kicks off in a football game. We often used the term, "It's a coin toss." Or "flip a coin." to describe events that are random with an equal chance of happening. In this activity students will investigate coin tossing as it relates to scientific data collection.

According to the Pascal triangle, if you toss 6 coins, you can get one of 7 possibilities with the percent probability of each listed:

Possibility	Outcome	Occurrence (64 tosses)	Percent
#1	0 heads, 6 tails	1	1.6%
#2	1 head, 5 tails	6	9.4%
#3	2 heads, 4 tails	15	23.4%
#4	3 heads, 3 tails	20	31.3%
#5	4 heads, 2 tails	15	23.4%
#6	5 heads, 1 tail	6	9.4%
#7	6 heads, 0 tails	1	1.6%

Based on the laws of probability and with fair coins, students should obtain the above outcomes. But the factor of random error will influence the results just as it does in scientific data gathering. Even so, it is obvious that it is very unlikely one would get 6 heads or 6 tails in a row.

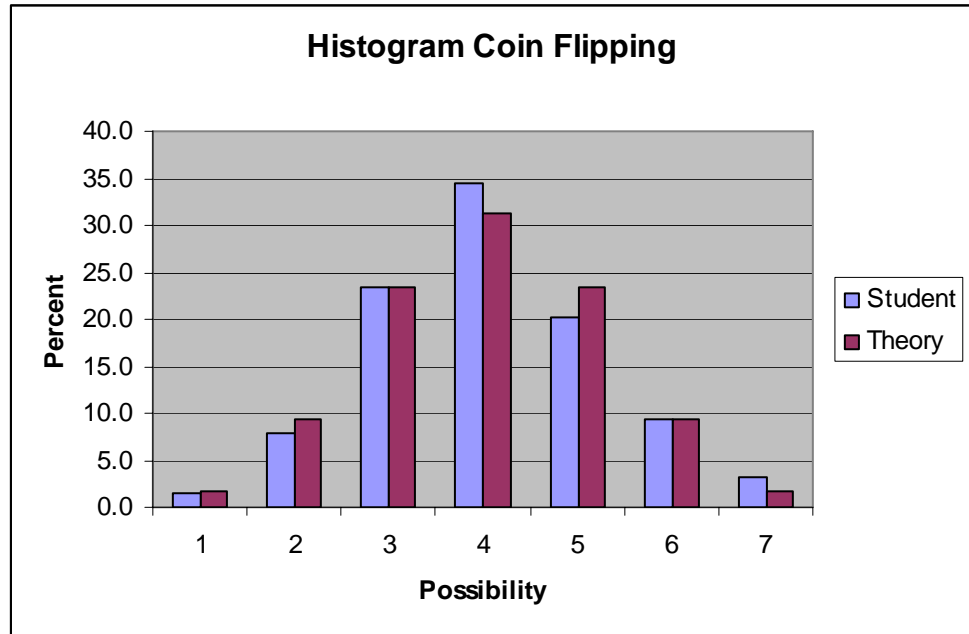
Procedure: The class will break into teams of two or three. Each team will have a box and lid with 6 pennies, and will do 64 trials. They will record their results by using hash marks in the Coin Flipping Table found at the end of this section. At the end of the 64 trials, complete the table by totaling each outcome. Calculate the percent you obtained for each possibility using the formula:

$$\text{Percent Occurrence} = \frac{\text{Occurance Total}}{\text{Total Trials}} \times 100$$

Make a histogram of the results and compare it with the mathematically predicted results. To do this, go to Excel. Input the Outcome Percent in column A and the theoretical percent in column B. Drag over these columns and select the graphing

icon. Choose Chart type: Column and click on the first box (Upper left corner). Complete the histogram and include it with your worksheet. A typical data table and histogram is shown below.

Student	Theory
1.6	1.6
7.8	9.4
23.4	23.4
34.4	31.3
20.3	23.4
9.4	9.4
3.1	1.6



How well did your coin flipping agree with Pascal's triangle?

Now compare results with other teams. How well did other teams do?

List the occurrences of each team on the board and in the table at the end of this section. Does random error affect the expected results? Remember that the measurements have a \pm random error which may affect a small data set.

To overcome the random error, scientists like to take lots of data. When this is done, it is hoped that the biases over a short run will cancel each other out. To see this, combine each of the students' groups into one large group. For example, combining 10 student teams will make 640 combinations. Does this grand sum more closely approach the expected occurrences predicted in the Pascal triangle? Can you see why this is so?

Complete the worksheet and hand in your tables and graphs.

Coin Flipping Activity Tables

Coin Flipping Table

Possibility	Outcome	Occurrence	Total Occurrences	Outcome Percent	Theoretical Percent
#1	0 H, 6 T				1.6%
#2	1 H, 5 T				9.4%
#3	2 H, 4 T				23.4%
#4	3 H, 3 T				31.3%
#5	4 H, 2 T				23.4%
#6	5 H, 1 T				9.4%
#7	6 H, 0 T				1.6%
Total Flips =					

Overall Coin Flipping Table

Outcome	Team 1	Team 2	Team 3	Team 4	Team 5	Team 6	Team 7	Team 8	Team 9	Team 10	Sum	Trial %	Theo. %
0 H, 6 T													1.6
1 H, 5 T													9.4
2 H, 4 T													23.4
3 H, 3 T													31.3
4 H, 2 T													23.4
5 H, 1 T													9.4
6 H, 0 T													1.6

Computer Simulation: It is time consuming and tedious to do the trials by hand when they can easily be done on a computer. Excel has a data analysis program with a random number generator and a histogram. (If not present on your computer, you will have to install it using the MS Office disk.)

In Excel, Under "Tools," select "Data Analysis" and "Random Number Generator."

- Number of Variables: 6
- Number of random numbers: 100
- Distribution: Binominal
- p value: 0.5
- Number of trials: 1
- Random seed: (blank)
- Select: Output range: \$A\$2 (Click on column A, row 2)

Hit "OK." You will see the individual coin flips in columns A to F.

Now sum up these results and use the sums as bins for the histogram.

- In column G, row 1, type in "Sum."
- In column G, row 2, select icon" Σ " and then the green check. Sum up all the rows in column G.

On the Excel sheet in a column I, enter "0", in row 1 enter "1", in row 2 enter "2"; etc until the column has 0, 1, 2, 3, 4, 5, and 6 in sequence.

Under "Tools," select "Data Analysis" and choose "Histogram"

- Input range: Drag over the "Sums"
- Bin range: Drag over the 0 to 6 column.
- Output range: \$J\$2
- Select "Chart Output" to get a histogram.

A histogram along with the bin of number of occurrences will be produced. Since you selected 100 trials, the occurrences are already in percent.

You can play with the chart and make it more presentable.

Teams can add their data together to see what happens when a large number of trials are taken. Likewise, a team may run a large number of trials themselves and compare it with the theoretical outcome.

4. Give a reason why you would expect the grand sum to be closer to the theoretical percentage.

5. Why do scientists like to take large data sets rather than small ones?

6. Are there any disadvantages to taking large data sets?

